

**USING FUZZY LOGIC FOR SOLUTION OF ECONOMIC TASKS:
TWO EXAMPLES OF DECISION MAKING UNDER UNCERTAINTY**

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Abstract

Paper is devoted to applying the principles and approaches of fuzzy logic and fuzzy modelling to the solution of applied economic problems in conditions of uncertainty. There are presents the formulation and formalization of the problem, the theoretical foundation of the proposed approach to the solutions, and the practical examples for two tasks of management and economic decisions making.

First is construction of investment portfolio. Diversification of investment portfolio as one of the approaches to risk management in the commercial activities in a competitive environment has been consider. Classical approaches to portfolio management are analyzed and practical difficulties of their application in real conditions of market uncertainty are identified. Corresponding methodological approach described and formulation for solution in fuzzy formulation is proposed. An example of numerical calculations in specialized software for operations with fuzzy numbers is given. The obtained results are described and analyzed.

Second is development of coordinated assessments of alternative choices for decision making in project management. The choice of alternative products, as part of project management, industrial and commercial solutions, has been described and analyzed. The necessity of the construction of generalized, mutually evaluations of projects, products or services on the basis of opinions of consumers, customers and manufacturers, has been justified. The use of the fuzzy sets for obtain such integrated assessments has been proposed. Mathematical formalized statement of the problem and calculations example are given.

Key words: *fuzzy modelling, decision-making in economy, investment portfolio construction, optimization of product programs, evaluation of alternative projects.*

JEL classification: C 02; C 60; D 81;

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1. INTRODUCTION

The aim of this paper is to propose and describe some approaches, based on fuzzy sets and fuzzy logic methodology, for solving of decision making problems and tasks in economy and management.

We have investigated the various tasks of decision-making in the economy in conditions of uncertainty. But in this paper we will consider only two of them, namely decision on the construction of investment portfolio and development of coordinated assessments of alternative choices for decision making in project management. Below we will detail look at each of these tasks.

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2. USING FUZZY APPROACH FOR DECISIONS FOR CONSTRUCTION OF INVESTMENT PORTFOLIO

As well known, the diversification of the investment portfolio can be viewed as one of the most important ways to manage investment risks.

One of the possible solutions for this problem is the Markowitz approach (Markowitz, 1959). Although it was widely used in the practice of portfolio management, however, this approach uses a number of assumptions which don't or bad correspond with the realities of investment processes.

2.1 Formulation of the problem and brief review of its decisions

It is obviously, that the statistical homogeneity condition cannot be practically achieved in real economic conditions and terms. Any economic entity or process, which is studied and analysed in the past time, and the same object at the moment – there are, in generally, two different objects of observation and investigation. Reasons for this are that the market environment of project had changes over time. Accordingly its market position will change.

So the risk of losses on a specific business direction will fall or rise, but the cause of these fluctuations will actually be in the external business environment. All these facts don't allow speak about statistical homogeneity of the studied process.

Appeal to subjective probabilities practically not improved the situation.

If probabilistic subjective assessment made by only one expert, when risk of subjectivity and erroneous forecast significantly increases (Kravets 1976). In fact, when expert applying the subjective probabilities, he refuses frequency approach and puts in the concept of probability his subjective expectations, which are largely based on the history and on the past experience. At the same time due to market conditions unplayable this prehistory ceases to be indicative and reliable for further the proved calculations and forecasting.

It is well known that the usage of correlation coefficients suggests the immutability for character of causal relationships. However, in actual economical practice this does not take place. Firstly, both the character and the relations themselves at the time of the study may be not fully defined. Secondly, there is the possibility of significant changes under the action of external forces and factors.

At the same time are known successful researches (Nowicki 2001; Nedosekin 2000) on the application of fuzzy sets for the construction of investment portfolios.

Further we proposed solution to this problem, which can be considered as a fuzzy analogue of statistical game.

For a start we will assume that there are a lot of various projects (investment projects, food programs, securities, etc.) from which is needed to build an investment portfolio. In other words it is necessary to properly distribute the investments in these projects.

We also assume that the information about the projects have vague, indefinite character, and at this stage of investigations its refinement due to the several time and some material costs. And take place the situation in which cannot be guaranteed to achieve the desired, required level of certainty.

The resulting estimates can be classified as expert and not always possible to view they in a quantitative form. In addition, quantitative estimates have expressed enough approximate nature. This problem formulated in fuzzy formulation can be solved in several ways (Dorokhov and Chernov 2011). At first we propose to consider the simplest way to solve it.

2.2 Proposed approach to consideration

Let suppose there are several projects of which is assumed to form an investment portfolio $S = \{s_j : j = \overline{1, N}\}$. We also assume known of estimates for possible incomes when performing (implementation) of these projects:

$$\tilde{C} = \left\| \tilde{C}_{i,j} \right\|. \quad (1)$$

Suppose also that we have constructed a set of possible combinations of these yields and there are identified expert assess of the probability of these combinations $\tilde{p}_i, i = \overline{1, M}$, where M is the number of considered combinations of component portfolio yields which forming a set of P :

$$\tilde{P} = \left\{ \tilde{p}_i; i = \overline{1, I_p} \right\} \quad (2)$$

In other words, for the relationship $\tilde{P}/S \quad S_1 \quad \dots S_n$ we have a matrix of probability estimates:

$$\begin{matrix} \tilde{P}_1 \\ \tilde{P}_2 \\ \tilde{P}_3 \\ \dots \\ \tilde{P}_m \end{matrix} \begin{pmatrix} \tilde{C}_{11} & \tilde{C}_{1n} \\ \tilde{C}_{21} & \tilde{C}_{2n} \\ \tilde{C}_{31} & \tilde{C}_{3n} \\ \dots & \dots \\ \tilde{C}_m & \tilde{C}_{mn} \end{pmatrix} \quad (3)$$

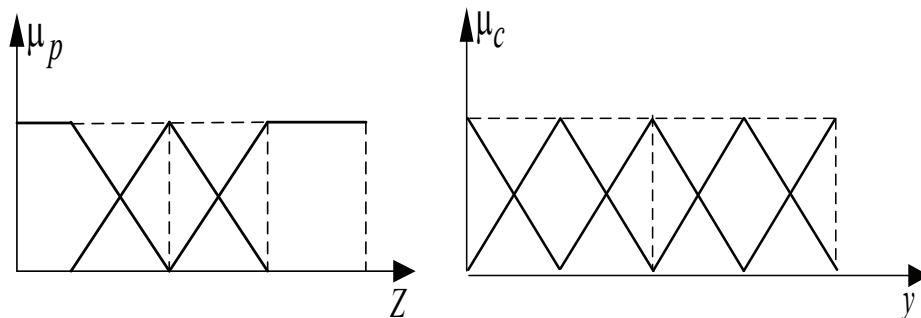
Sets (1) and (2) can be associated with linguistic assessment of their possible values. Thus, for the possibilities for implementing of P we can consider $L_p = (l_{p_i} : i = \overline{1, I_p})$, $M_p = \{ \mu_{l_{p_i}}(z) / z \}$, and respectively, for yield C we can consider: $L_c = \{ l_{c,j} : j = \overline{1, J_c} \}$ and $M_c = \{ \mu_{c,j}(y) / y : j = \overline{1, J_c} \}$.

For simplicity, we assume that $z \in [0,1]$ и $y \in [0,1]$. For variable z this condition is quite natural, since we are talking about probabilities. For variable y it belongs to the interval $[0,1]$ indicates the need to pass from absolute to relative scale. For the simplify the presentation let us assume that the set L_p consists of 3 elements:

$L_p = \langle \text{small, medium, large} \rangle = \langle S, M, B \rangle$ (as shown in Fig. 1, left), and correspondingly L_c consists of 5 elements:

$L_c = \langle \text{small, less than medium, medium, more than medium, large} \rangle = \langle S, LM, M, MM, B \rangle$ (as shown in Fig. 1, right).

Figure 1. Membership functions for linguistic probability estimates (left) and yield (right)



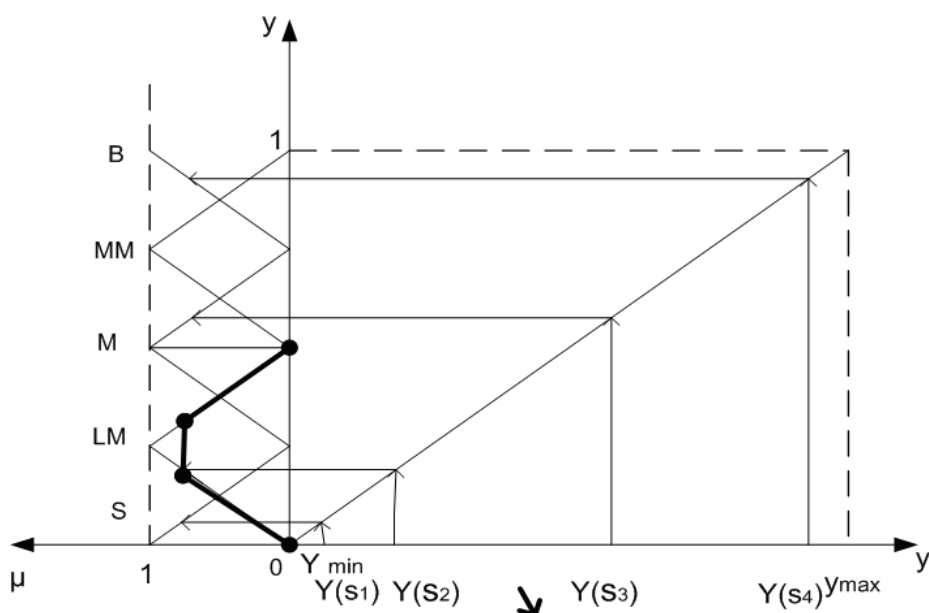
Source: authors' own researches

Triangular membership functions (Fig. 1) were chosen for reasons of simplicity and only for elementary graphical representation. Estimates presented in Fig. 1 can be constructed in various ways. Consider some of them on the assessments yield.

The first variant is that the investor's experts indicate the boundaries of possible yield $[y_{\min}, y_{\max}]$. Then, within this interval we can defined subintervals, which, according to experts opinions we can match the selected linguistic estimates, such as those indicated in Fig. 1. If for the solution of the problem has been developed computer software and corresponding information system, then interactively can be constructed appropriate membership functions on the respective subintervals based on established rules (Dorokhov and Chernov 2011). They can be defined either by an expert, or selected from a set of proposed by mentioned software system. After that, it is converted to a relative scale.

Another variant may be illustrated by the following figure (Fig. 2).

Figure 2. Graphical method of transition from the numerical estimates of yield for components of an investment portfolio to linguistic assessments.



Source: authors' own researches

In this figure $Y(s_i)$ – is the numerical assessment of yield for i - th component of an investment portfolio.

For each $i=1,2,3,4$; $Y(s_1) \rightarrow S$, $Y(s_2) \rightarrow LM$, $Y(s_3) \rightarrow M$, $Y(s_4) \rightarrow \hat{A}$ - is the linguistic evaluation for components of portfolio (thick lines shows a membership function for $Y(s_2) \rightarrow LM$).

In general case the conversion function can also be nonlinear. As a result, the matrix (3) will contain only the linguistic evaluations for a four component portfolio. It may be, for example, as show follows in Table 1.

Table 1 – The numerical example of utility matrix for a four components portfolio (the calculations are done in an special environment for operations with fuzzy numbers and variables named Fuzicalc)

	A	B	C	D	E	F	G
53		probability	assessments of profitability for components of the investment portfolio				
54		P/S		S1	S2	S3	S4
55		XXXXXXXXXX	XXXXXXXXXX	XXXXXXXXXX	XXXXXXXXXX	XXXXXXXXXX	XXXXXXXXXX
56		S	*	B	MM	B	B
57		S	*	B	B	MM	MM
58	W=	M	*	M	MM	MM	M
59		M	*	M	S	M	S
60		B	*	S	S	M	M
61		B	*	S	M	M	S
62							

Source authors' own researches

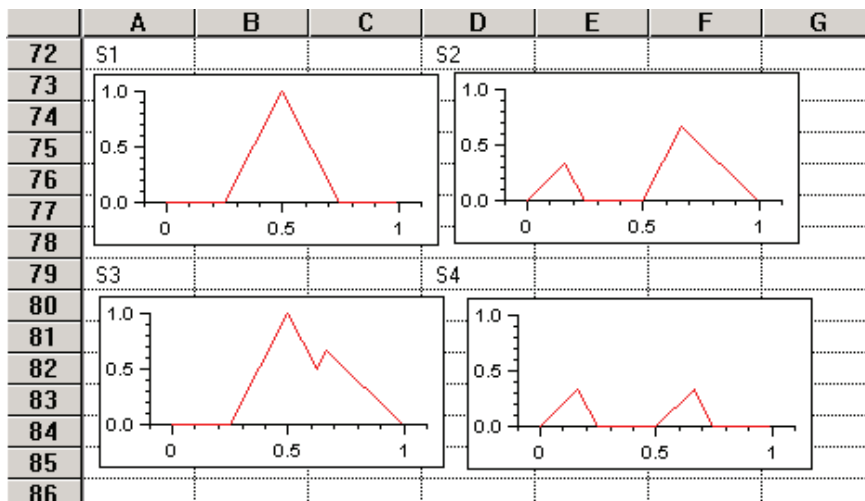
2.3 The solving of the problem

Solution of the problem consists in determining the coefficients of proportionality for components of the investment portfolio, can be carried out as follows. For each column $S_j, j = \overline{1, N}$ can be calculated $\tilde{w}_{i,j} = \tilde{p}_i \wedge \tilde{c}_{ij} = \min(\tilde{p}_i, \tilde{c}_{ij}), i = 1, 2, \dots, m, i \in [1, m]$. Then we can find (Fig. 4):

$$\tilde{w}_j = \bigcup_i \tilde{w}_{ij} = \max_i [\tilde{w}_{ij}] \quad (4)$$

For each component of the planned investment portfolio S_j can be estimated \tilde{w}_j – this is the fuzzy set with the corresponding membership function (Fig. 3).

Figure 3. The resulting membership functions



Source: authors' own researches

Aggregate of \tilde{w}_j can be considered as the vector of priorities $\tilde{W} = \{w_j : j = \overline{1, N}\}$, by which is possible to evaluate the distribution of funds to components of portfolio.

However, the direct use of the vector \tilde{W} is plenty difficult, so it is possible to use the following variant. For all fuzzy sets \tilde{w}_j is calculated coordinate of the center of gravity of the figure bounded by the membership function $\mu_{\tilde{w}_j}(z)$ by the following formula:

$$CG_j = \frac{\sum_i \mu_{\tilde{w}_j}(z_i) z_i}{\sum_i \mu_{\tilde{w}_j}(z_i)} \quad (5)$$

As a result, we can get not normalized vector of priorities $CG^{\bar{}} = \{CG_j : j = \overline{1, N}\}$, which may be converted into a normalized, if we calculate the values:

$$CG_j^h = \frac{CG_j}{\sum_j CG_j} \quad (6)$$

These values can be considered as the coefficients of proportionality for the allocation of funds for components of the investment portfolio.

2.4 Account of the multiplicity of identical ratings

As obviously considered method does not account for the multiplicity of identical ratings. However it is clear that the alternative which has a larger number of identical high ratings (of course, if there are not losses) is more preferable than other alternatives with fewer number of these estimates or more number of lower estimates.

Take into account this multiplicity we can as follows. After calculating the values CG_j^i must be determined the multiplicity of the same estimates located on the left and right of the average estimate. Then is calculated the coefficient of multiplicity:

$$K = 1 + \frac{k_R - k_L}{k_L + k_R}, \quad (7)$$

where k_L – is the multiplicity of the same estimates on the left of the average estimate; k_R – is the multiplicity of the same estimates on the right of the average estimate.

Taking into account multiplicity coefficient, we can to calculate the modified value as:

$$CG_j^M = K * CG_j^i. \quad (8)$$

Obviously, if the prevailing right estimations, the value of CG_j^i is shifted in this direction and $K > 1$. Predominance of left estimations corresponds to the value $K < 1$, that defines a corresponding shift of CG_j^i .

Table 2 shows the results of calculations for the matrix of linguistic estimates presented in Table 1. Calculations were performed as without taking into account factor of estimates multiplicity and well as with taking them into account. The results are quite consistent with the character of the initial data and the estimates obtained by the relation (4) and given in Table 1 and Figure 2.

Note the effect of the multiplicity of similar estimates, which led to an increase in the coefficient of proportionality of the third component of the investment portfolio. It also is quite consistent with the initial data.

Research on the influence of the form of the membership function on the final results showed that this effect takes place. However, in both cases (in the absence of accounting for multiplicity of estimates, and when it is taken into account) changes in ownership interests in a portfolio have not a cardinal character.

Table no. 2 – The calculation results for the matrix of linguistic evaluations
(calculations were made in Fuzicalc environment)

	A	B	C	D	E	F	G	H
87		EffPeak						
88		0.5	0.743443	0.695654	0.23989			
89	coefficients of proportionality for components of the investment portfolio							
90		0.229464	0.341188	0.319256	0.110092			
91	Kl[s1]=	3	Kl[s2]=	1	Kl[s3]=	0	Kl[s4]=	2
92	Kr[s1]=	2	Kr[s2]=	3	Kr[s3]=	3	Kr[s4]=	2
93	KK=	0.8		1.5		2		1
94	EffPeak*KK	0.4		1.11517		1.39131		0.23989
95	coefficients of proportionality for components of the investment portfolio with multiplicities of ratings							
96		0.127131		0.35443		0.442196		0.0762436
97		0.1		0.35		0.47		0.08

Source: authors' own researches

The disadvantage of the proposed method is the use of the operation of intersection in the construction of fuzzy sets \tilde{w}_j . It is really possible situation, when some $\tilde{w}_{ij} = \emptyset$, or in an extreme case for some j -th component of portfolio all $\tilde{w}_{ij} = \emptyset$. Nevertheless, this result does not mean that the corresponding component of the portfolio should unequivocally be deleted from further consideration.

2.5 Definition of operation for build the shadow of fuzzy sets

To resolve this situation instead of the operations of intersection can be used other operation, proposed in (Chernov, 2007; Chernov, 2010) and defined as the shadow of a fuzzy set $Sh(\tilde{A}, \tilde{B})$. Shadow of a fuzzy set \tilde{A} to fuzzy set \tilde{B} must satisfy the following requirements: $Sh(\tilde{A}, \tilde{B})$ - fuzzy set; $Sh(\tilde{A}, \tilde{A}) = \tilde{A}$; $Sh(\tilde{A}, \tilde{A}) = \emptyset$, if at least one of the sets \tilde{A} or \tilde{B} is empty or the sets \tilde{A} and \tilde{B} are orthogonal.

The procedure for constructing the shade of a fuzzy set \tilde{A} to fuzzy set \tilde{B} can be defined as follows (Fig. 4):

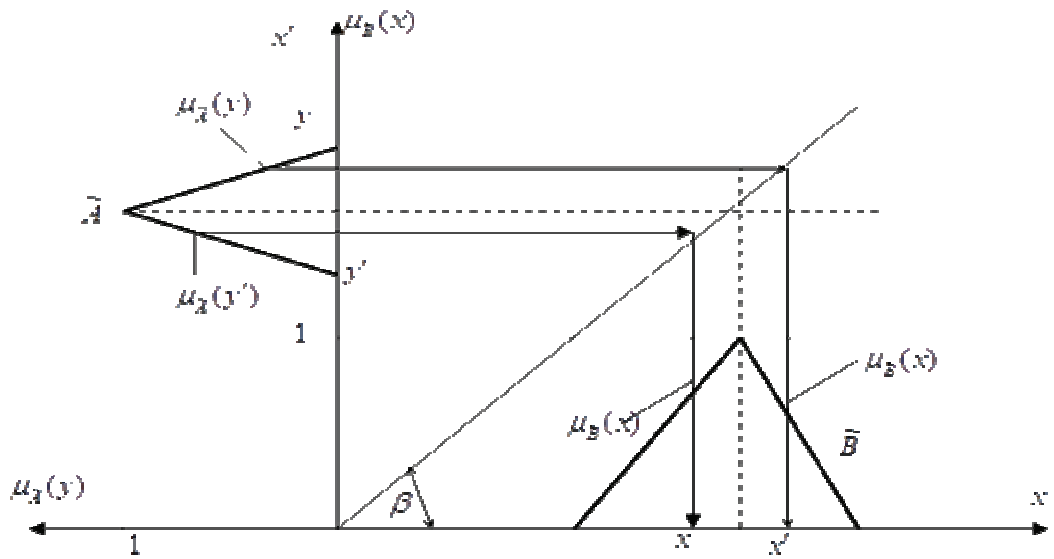
$$Sh_{\varphi}(\tilde{A}, \tilde{B}) = \{\varphi[\mu_{\tilde{A}}(y), \mu_{\tilde{B}}(x)]/[y, x' = f(y)]\},$$

where $f(y) = \frac{CG[\mu_{\tilde{B}}(x)]}{CG[\mu_{\tilde{A}}(y)]}y$ - is the projection function;

$CG[\mu_{\tilde{B}}(x)], CG[\mu_{\tilde{A}}(y)]$ - coordinates of the centers of gravity of the figures which are bounded by the membership functions $\mu_{\tilde{A}}(y)$ and $\mu_{\tilde{B}}(x)$;

φ - is the functional which sets the way for the transformation of membership functions.

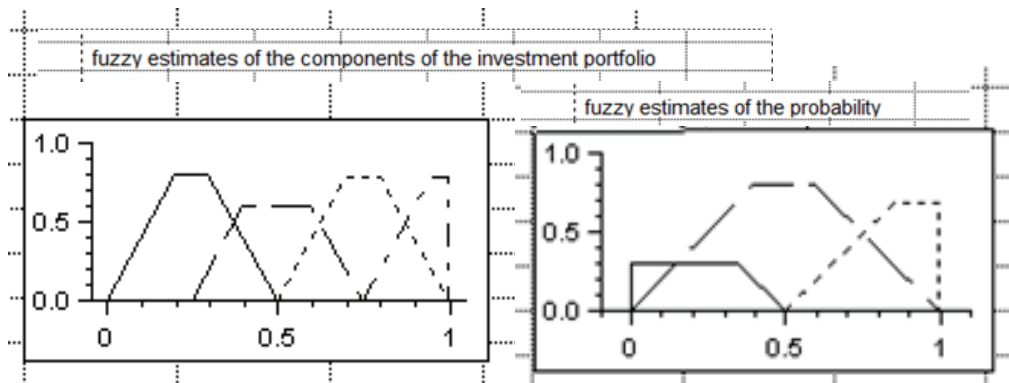
Figure no. 4 The geometric representation of the build operation for the fuzzy set shadow



Source: authors' own researches

Next (in Fig. 5) shows the membership functions of fuzzy estimates of the probability and the components of the investment portfolio, which were used in further calculations.

Figure 5. Geometric representation of the build operation for the fuzzy set shadow



Source: authors' own researches

2.6 Calculations using the shadows of a fuzzy sets

Sequence of computations using the operations using the shade of a fuzzy set is as follows. For each column $S_j, j = \overline{1, N}$ calculated:

$$\tilde{w}_{i,j} = Sh(\tilde{p}_i, \tilde{c}_{i,j}) = \{\varphi[\mu_{\tilde{p}_i}(y), \mu_{\tilde{c}_{i,j}}(x)]/[y, x' = f(y)]\}, \quad (9)$$

where $i = 1, 2, \dots, m, i \in [1, m]$;

$$f(y) = \frac{CG[\mu_{\tilde{c}_{i,j}}(x)]}{CG[\mu_{\tilde{p}_i}(y)]} y \quad \text{- is the projection function, } x \in [0, 1], y \in [0, 1];$$

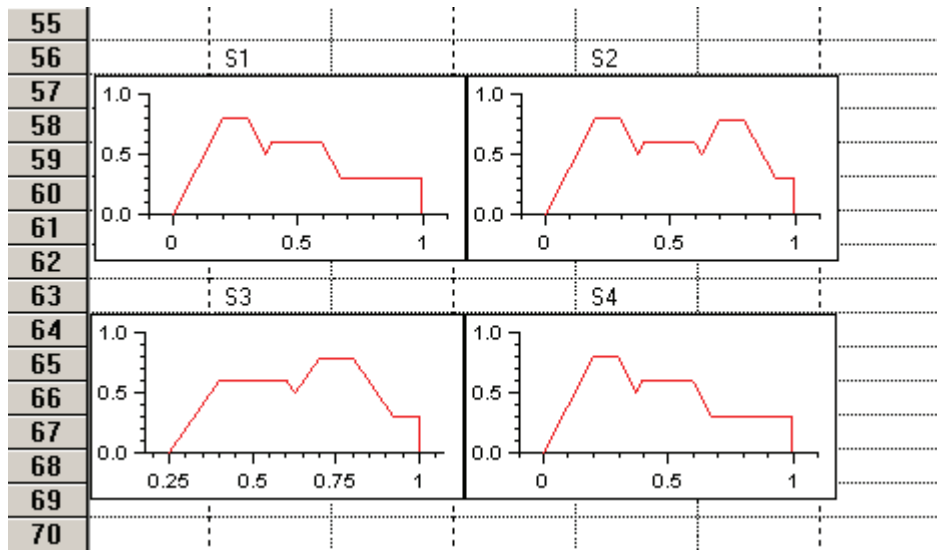
$CG(\mu_{\tilde{c}_{i,j}}(x)), CG(\mu_{\tilde{p}_i}(y))$ - are the coordinates of the centers of gravity of the figures which are bounded by the membership functions $\mu_{\tilde{p}_i}(y), \mu_{\tilde{c}_{i,j}}(x)$;

$\varphi = \min$ - is the used functional which give specifying transformations over the membership functions.

$$\text{Then is finded } \tilde{w}_j = \bigcup_i \tilde{w}_{ij} = \max_i [\tilde{w}_{ij}].$$

Results of transformations according to Table 1 and Fig. 5 with using the relations (9) are shown in Fig. 6.

Figure 6. Membership function of resulting estimates for investment portfolio components



Source: authors' own researches

Further calculations are carried out according to equations (6-8) in program tools Fuzicalc, and it's the results are given in Table 3.

Table 3. The calculation results for the matrix of linguistic evaluations (calculations have been made in an environment Fuzicalc)

coefficients of proportionality for components of the investment portfolio			
S1	S2	S3	S4
0.221205;	0.246747;	0.310843;	0.221205;
coefficients of multiplicity of ratings			
0.8;	1.5;	2;	1;
Коеф. coefficients of proportionality for components of the investment portfolio with multiplicities of ratings			
0.127556;	0.266205;	0.44714;	0.159099;
(0.13);	(0.26);	(0.45);	(0.16);

2.7 Analysis of the results

Comparison of the results given in Table 2 and Table 3 shows their obvious qualitative coincidence. The difference in the numerical values can be explained by the fact that the use of operations *Sh* allows more fully take into account all aspects of decision-making situation for investment portfolio.

The proposed method of investment portfolio formation is not only free from the shortcomings of other known methods. It also gives the opportunity (unlike other methods) to take into account the multiplicity of identical estimates of yield component of the investment portfolio.

Described method also allows increase the validity of the allocation of resources. In practice, this method has been successfully applied in the evaluation of the various options for portfolios in a few Ukrainian commercial banks and investment funds.

3. DEVELOPMENT OF COORDINATED ASSESSMENTS OF ALTERNATIVE CHOICES FOR DECISION MAKING IN THE PROJECT MANAGEMENT

Let us consider the second task mentioned and posed in beginning of the paper. Among the tasks of project management a great place devoted to the analysis, optimization, designing of rational and effective product programs. This is particularly important in a competitive market environment in all sectors of material production and service delivery in Ukraine.

Open, globalized economy requires proper positioning of any economic entity in the market, in particular, error-free, science-based, timely and effective range of issues, ratio of various products - as that which is produced and planned to start of production, and the corresponding analysis and the actions of competitors and of the environment (Boc *et al*, 2012; Ondruš *et al*. 2013). A powerful tool for this purpose is a multicriteria analysis of the alternative production programs.

3.1 Approaches to building a multilateral assessments for product programs

It is clear, that the most acceptable and desirable is the situation, when the better product programs is given thoughts, wishes and economic and social aspirations as a manufacturer and designer, and consumer guides, buyer of the goods, project, services. Thus there are multidimensional criteria for the problem of selection.

The validity of the results of solving this problem of multi-alternative choice can be significantly enhanced if the process will be considered methodologically correct valuation and preferences of several parties. On the one hand, there are experts, representing the interests of firms, enterprises, and possible conflicts in their assessments are that they have different can evaluate compliance products selected criteria, as well as in different ways can come to the criteria and degree their importance, however, in general their actions and assessments have the same orientation. But the situation is much more complicated, if the one or more experts are the users or customers. Alignment of assessments with each other is pointless, unless this desire is not wishful thinking.

Instead, mutual attitudes of users and experts representing the interests of manufacturers, it is useful because it can be built together a balanced system of assessments. Then the product that meets it, will be the best way to meet both restrictions and willingness, capacity to manufacturer requirements and preferences of the user, that is, in general - today's modern competitive market conditions.

The investigated problem is compounded by the fact that preferences and advantages of the parties are likely to be fuzzy, especially when it comes to the new or significantly new products. In this case, it is supported by the theory of fuzzy sets which allows to achieve reasonable and adequate results (Mcneil *et al*, 1994; Da Ruan, 2000; Bojadziev *et al*, 2007; Wang *et al*. 2007).

3.2 Formulating the goals and description of the main research

According to the methodology of fuzzy modeling such problems can be resolved on the basis of the composition of fuzzy criterion evaluations and heuristic reasoning as a linguistic assessments of utility, this problem is formulated in general terms in (Efstathion 2011). The corresponding model of the decision-making process includes: assessment alternatives; formalized opinions of persons, that take part in the decision-making process; description of the process of decision-making.

At the same time each alternative product can be described by quality criteria, the values of which are often provided by the interviewees in fuzzy linguistic form and can be expanded by the modifier “very”, “more or less” and so on (Dorokhov and Chernov 2011). Accordingly, the aim of the researches is the mathematical formulation and implementation of practical methods of obtaining coordinated assessments of alternative products, projects or production programs using instruments of fuzzy set theory.

Look at the formulation and solution of this problem in a summarised form.

Let used N criteria as $D_i, i = \overline{1, N}$, which form space of criteria $D = D_1 \times D_2 \times \dots \times D_N$, any point of which is specified by a set of values $d = \{d_1, d_2, \dots, d_N\} \in D, d_i \in D_i, i = \overline{1, N}$.

Opinions of people, that take part in the decision-making, can be interpreted as vague subspaces space criteria $D: W = F(D_1) \times F(D_2) \times \dots \times F(D_N)$, where $F(D_i)$ – is fuzzy subset D_i .

The fuzzy membership function for composition W can be defined as:

$$\mu_w(d_N) = \min \mu_w(d_i), i = \overline{1, N}, \quad (10)$$

where $\mu_w(d_i)$ – is the fuzzy membership function of W by criterion D_i .

The usefulness of appropriate criteria can be defined as a linguistic variable whose value is a term of fuzzy set defined on interval $[0,1]$.

Interaction between criteria and usefulness is determined, as fuzzy relation $R: d \rightarrow U$, which exists on $D \times U$. Usually R are represented in the table, which sets usefulness for different points in the space of criterion D . Representation of the usefulness in the form of fuzzy relation R allows us to determine W fuzzy subsets $V \subset U$ as composition ratio:

$$\mu_V(z) = \max\{\min[\mu_R(d_n, z), \mu_w(z)]\}.$$

According to definition of V , each composition W can have more than one value of usefulness and all of them are characterized by different degree of membership.

Usefulness with the highest degree interior can be accepted as the basis for comparison of different compositions of W .

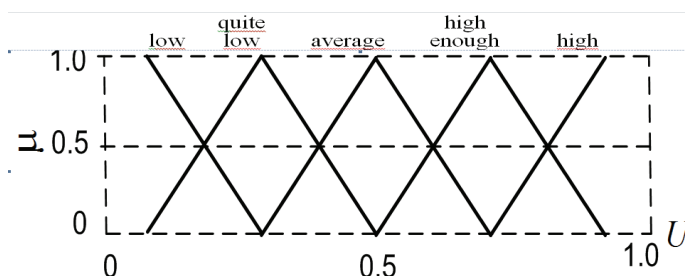
Suppose that the decision to release a new product has been formed by two groups of experts: the first group includes the employees of the manufacturer, and the second group includes potential customers, buyers, users of the products.

The process of group formation is beyond the scope of our research and is not considered. The only requirement is to provide experts assessments, which do not conflict with common sense.

Both groups give their product evaluations, which form set of efficiency criteria and goals. Let denote this set as: $D = \{d_i; i = \overline{1, N}\}$. For each d_i has been defined plural linguistic assessments: $L(d_i) = \{l_{j(d_i)}; j(d_i) = \overline{1, M(d_i)}\}$.

Number of linguistic estimations and its formulation depend on specific of assessments and their criteria. If necessary, can be used logical identifiers. In the process of work with the expert groups can build a plural linguistic assessments for usefulness, for example, as represented in Figure 7, as well as can be built heuristics of experts $E_1 = \{\varepsilon^1_k : k = \overline{1, K}\}$ and heuristics of consumers $E_2 = \{\varepsilon^2_q : q = \overline{1, Q}\}$.

Figure 7. Sets of membership functions described the linguistic assessments for usefulness



Source: authors' own researches

For example, are possible such heuristics as "fuel consumption may be slightly more if the car will have a high permeability" or "tire noise can be quite high if provided good traction in all weather conditions". By appropriate analysis heuristics we can determine with the assessment of the product that best agreed with the view of both sides of the expert evaluation. The initial, full space of efficiency D contains P_j sets $M(d_i)$, $i = \overline{1, N}$.

Through further subsequent analysis of heuristics by experts it can be rejected all insignificant estimations for both groups criterion for evaluating the effectiveness and continue to get refined cut sets criteria values: $D' = \{d'_i; i = \overline{1, N}\}$.

According to this method can be achieved reduction in evaluative expert sets that cover the entire region of utility. After that, both groups of experts must find agree on joint, coordinated assessments benefits of new joint sets. For example, can be built next Table 4.

Table 4. The general view of the data source information.

Criteria	Usefulness for the consumers	Usefulness for the experts
D₁ D₂ ... D_N		
Set 1	Linguistic evaluations	Linguistic evaluations
Set 2		
...		
Set M		

For greater certainty we specify further consideration of assessment, leaving abstract form criteria. The final results can be represented in form of Table 5, where we are limited to four criteria in abstract form.

Table 5. The linguistic assessment for usefulness of sets based on the criteria of consumers and experts.

Sets	Criteria (its level and linguistic assessment)				Usefulness for the consumer	Usefulness for the experts
	D₁	D₂	D₃	D₄		
Set 1	excellent	very high	big	substantial	very, very high	very, very high
Set 2	good	high	enough	substantial	very high	very high
Set 3	excellent	acceptable	big	small	high enough	high
Set 4	excellent	weak	big	small	weak	the average
Set 5	good	acceptable	big	small	the average	below average
Set 6	good	weak	big	small	weak	below average

For further consideration of practical example let consider numerical evaluations.

3.3 Example of numerical calculations according to proposed algorithm

We will consider the case, when there are N-alternative products, $P = \{P_n: n = 1, N\}$.

For each of the criteria of effectiveness we will built corresponding fuzzy set $(D_i) = \{a_h / d_i, i = 1, \dots, N\}$, $a_h \in [0,1]$, where a_h is the assessment of compliance with h-th product and i-criteria.

For example, for some product the condition, that his assessment criterion D_1 closer to “good”, than to “great”, will be marked as: $F_h(D_1) = (0,3 / \text{great}, 0,7 / \text{good})$. Similar assessments can be built for all criteria.

For example, we will assume, that:

$F_h(D_2) = (0,6 / \text{acceptable}, 0,4 / \text{weak})$,

$F_h(D_3) = (0,4 / \text{enough}, 0,6 / \text{big})$,

$F_h(D_4) = (1 / \text{small})$.

Direct multiplication for $F_h(D_i)$ with all $i = 1, \dots, N$ gives all possible combination of the assessment product h to chosen criteria and the corresponding value of membership functions are determined by (1).

Then alternative A_h will gets the following values:

0,3 / (excellent, acceptable, enough, small),

0,3 / (excellent, weak, enough, small),

0,3 / (excellent, acceptable, big, small),

0,3 / (excellent, weak, big, small),

0,4 / (good, acceptable, enough, small),

0,4 / (good, weak, enough, small),

0,6 / (good, acceptable, big, small),

0,4 / (good, weak, big, small).

In further calculations, we should note and take into account the identical combinations of estimations. Accordingly, we can to obtain estimates of the considered alternatives in view of its usefulness to the consumer:

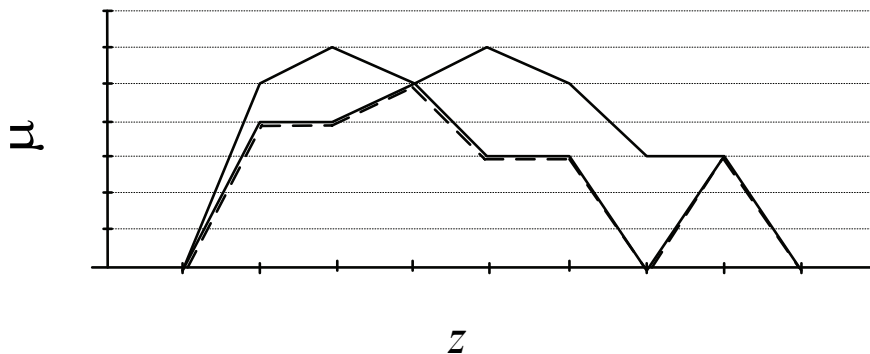
$A_{h_n} = \{0,3 / \text{high enough}; 0,3 / \text{weak}; 0,6 / \text{average}; 0,4 / \text{weak} \}$.

In its turn, for the experts we will have:

$A_{h_3} = \{0,3 / \text{high}; 0,3 / \text{average}; 0,6 / \text{below average}; 0,4 / \text{below average} \}$.

Linguistic assessments (linguistic evaluations), used in mathematical expressions for A_{h_n} and A_{h_3} , are shown in the on Figure 8.

Figure no. 8 Resulting linguistic evaluation of usefulness for A_{h_n} and A_{h_3} .



Source: authors' own researches

Let assume that the numerical values for the corresponding membership functions have next view:

High enough = {0/0,0/0.1,0/0.2,0/0.3,0/0.4,0/0.5,0.5/0.6,1/0.7,0.5/0.8,0/0.9,0/1.0};
 Weak = {0/0,0/0.1,0.5/0.2,1/0.3,0.5/0.4,0/0.5,0/0.6,0/0.7,0/0.8,0/0.9,0/1.0};
 The average = {0/0,0/0.1,0/0.2,0/0.3,0.5/0.4,1/0.5,0.5/0.6,0/0.7,0/0.8,0/0.9};
 High = {0/0,0/0.1,0/0.2,0/0.3,0/0.4,0/0.5,0/0.6,0/0.7,0.5/0.8,1/0.9,1/1.0}.

The resulting, final evaluation on the basis of partial assessments will determine by the minimum approach.

Then $A_{kn} = \{ [0/0, 0/0.1, 0/0.2, 0/0.3, 0/0.4, 0/0.5, 0.3/0.6, 0.3/0.7, 0.3/0.8, 0/0.9, 0/1.0];$
 $[0/0, 0/0, 0.3/0.2, 0.3/0.3, 0.3/0.4, 0/0.5, 0/0.6, 0/0.7, 0/0.3, 0/0.9, 0/1.0];$
 $[\{ 0/0, 0/0.1, 0/0.2, 0/0.3, 0.5/0.4, 0.6/0.5, 0.5/0.6, 0/0.7, 0/0.8, 0/0.9 \};$
 $[0/0, 0/0.1, 0.4/0.2, 0.4/0.3, 0.4/0.4, 0/0.5, 0/0.6, 0/0.7, 0/0.8, 0/0.9, 0/1.0] \}.$

Using more principle of maximum we also can to obtain:

$A_{kn} = \{0/0, 0/0.1, 0.4/0.2, 0.4/0.3, 0.5/0.4, 0.6/0.5, 0.5/0.6, 0.3/0.7, 0.3/0.7, 0.3/0.8, 0/0.9, 0/1.0\}$

Similarly, for experts:

$A_{k3} = \{ [0/0, 0/0.1, 0/0.2, 0/0.3, 0/0.4, 0/0.5, 0/0.6, 0/0.7, 0.3/0.8, 0.3/0.9, 0.3/1.0];$
 $[0/0, 0/0.1, 0/0.2, 0/0.3, 0.3/0.4, 0.3/0.5, 0.3/0.6, 0/0.7, 0/0.8, 0/0.9, 0/1.0];$
 $[0/0, 0/0.1, 0.5/0.2, 0.6/0.3, 0.5/0.4, 0/0.5, 0/0.6, 0/0.7, 0/0.8, 0/0.9, 0/1.0];$
 $[0/0.1, 0/0.1, 0.4/0.2, 0.4/0.3, 0.4/0.4, 0/0.5, 0/0.6, 0/0.7, 0/0.8, 0/0.9, 0/1.0] \}.$

And finally:

$A_{k3} = \{0/0, 0/0.1, 0.5/0.2, 0.6/0.3, 0.5/0.4, 0.3/0.5, 0.3/0.6, 0/0.7, 0.3/0.8, 0.3/0.9, 0.3/1.0\}.$

Finally, joint estimation of consumers and experts define a as crossing $\mu_R(z) = \mu_A^3(z) \wedge \mu_A^n(z) = \{0/0, 0/0.1, 0.4/0.2, 0.4/0.3, 0.5/0.4, 0.3/0.5, 0.3/0.6, 0/0.7, 0.3/0.8, 0/0.9, 0/1.0\}.$

3.4 The consideration of obtained results

At the next, final stage of the analysis raises the question of consistency of opinions of customers and experts. One alternative for estimating the degree of consistency is, for example, the Hamming distance:

$$S = \sum_i S_i, \text{ where } S_i = \max [\mu_A^3(z_i), \mu_A^n(z_i)] - \min [\mu_A^3(z_i), \mu_A^n(z_i)].$$

In case with the agreed of opinions, $S \rightarrow \min$. For our example, $S = 1.7$

However, the full rate of estimation the alternatives closeness by Hamming distance in this case is quite difficult. It is unclear concerning what assess this value. More certain is the assessment of the level of weighted power of the respective fuzzy sets. Indeed, at similar views:

$$S_A^3 \approx S_A^n, \text{ where } S = \sum S_i, S_i = \sum_j X_j / n_j \partial \alpha_j; \text{ where } \alpha_j - j\text{-a level } \partial \alpha_j = \alpha_j - \alpha_{j-1},$$

X_j – is the value argument supplies, is that functionality $\mu(x_i) \geq \alpha_j$;

n_j – is the number of X_j .

For considered example we will have: $S_A^3 = 0.41$, $S_A^n = 0.29$.

The degree of the coherence of experts opinions in this case is: $\xi = S_A^3 / S_A^n = 0.29 / 0.41 = 0.7$, that is, it is high enough.

We mention one conclusion that can be drawn from the obtained result. Let see Figure 9, where are shown together graphics for evaluate of the usefulness and the resulting membership function $\mu_R(z)$.

Onward we will denote the membership and useful functions as $\mu_n^i(z)$, $i = 1, 2, 3, 4, 5$.

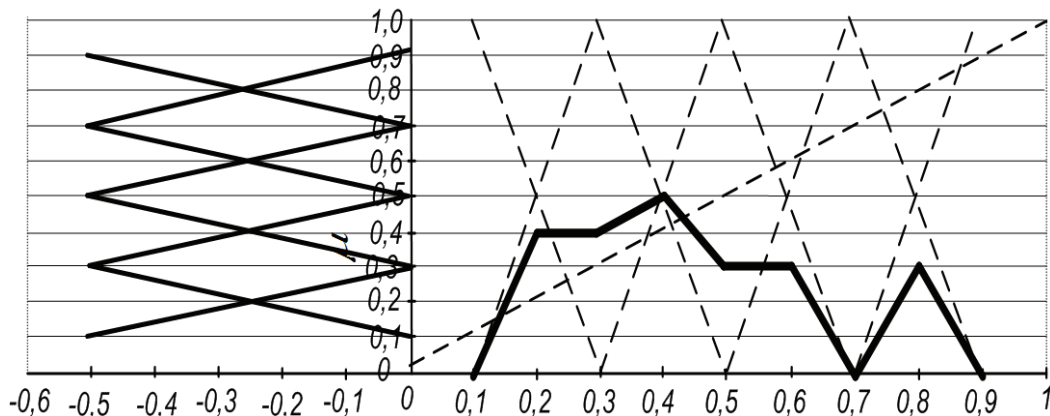
Let's build a crossing of functions $\psi_i \mu_n^i(z) \wedge \mu_R(z)$, $i = 1, 2, 3, 4, 5$.

Each ψ_i can be interpreted as a fuzzy set of matches joint assessment of many consumers and experts for some value of usefulness.

For comparison of ψ_i between we can to calculate their capacity through number of elements for which $\mu(z) \geq \alpha_{ij}$. In this case we will use the ratio $S_{\alpha ij} = n_{\alpha ij} \partial \alpha_{ij}$, $\partial \alpha_{ji} = \alpha_{ji} - \alpha_{(j-1)*i}$, $S(\psi_i) = \sum_{ji} S_{\alpha ij}$

In addition we will calculate the total power $S(\psi) = \sum_i S(\psi_i)$.

Figure 9. Evaluation of the usefulness and the resulting membership function



Source: authors' own researches

For this example we will obtain: $S(\text{low}) = 0.8$; $S(\text{quite low}) = 2.7$; $S(\text{average}) = 2.1$; $S(\text{high enough}) = 1.1$; $S(\text{high}) = 0.4$.

It is obvious that the most reasonable are the usefulness concept "quite low" and "average". The weights of these evaluations $V_i = S(\psi_i) / S(\psi)$ are respectively 38% and 29.5%. Thus, the obtaining harmonized evaluation of product in this case suggests unreasonableness or low expediency of its production.

So, we have proposed and described (simple but enough yet reasonable by estimates from the sidelines as producers and consumers of products or services) the algorithm based on fuzzy multiple approaches to the treatment of consumer market preferences expressed in linguistic form.

It allows us to capture and analyze the generalized opinion on both sides of the market (producers and consumers) as integral indicator for usefulness of certain goods, services, and products. This approach can be useful and quite successfully applied to develop rational management and marketing solutions in a competitive market environment.

4. CONCLUSIONS

In this paper are described, solved and analyzed two multicriteria decision making tasks with usage fuzzy logic approach.

First task deals with decision on the construction of investment portfolio.

Diversification of the investment portfolio as one of the approaches to risk management in the industrial and commercial activities in a competitive environment has been considered.

Classical approaches to portfolio management are analyzed and practical difficulties of their application in real conditions of market uncertainty are identified. The corresponding the methodological approach has been described and formulation for solution of the problem in fuzzy formulation is proposed. An example of numerical calculations implemented in specialized software for operations with fuzzy numbers is given. The obtained results are described and analyzed.

The second was development of coordinated assessments of alternative choices for decision making in project management. The problem of choice of alternative products or production programs, as part of project management, industrial and commercial solutions, has been described and analyzed.

The necessity of the construction of generalized, mutually evaluations of projects, products or services on the basis of opinions of consumers, customers and manufacturers, has been justified.

The use of the fuzzy sets theory for obtain such integrated assessments has been proposed. Appropriate mathematical reasoning, formalized statement of the problem, the sequence of its solutions and calculations example were given.

Summarizing the results obtained it can be stated the possibility and expediency of the use of approaches based on fuzzy sets theory in the study of decision-making problems in the economy in the conditions of uncertainty.

The authors suggest continue these studies, in particular, for the problem of logistics and modelling the interaction of consumers and producers of products and services in a competitive market environment.

REFERENCES

- Boc, K., Vaculík, J. and Vidriková, D. (2012), "Risk analysis in managerial process and fuzzy approach", *Transport and Telecommunication*, 13(1), 214–222.
- Bojadziev, G. and Bojadziev, M. (2007), *Fuzzy Logic for Business, Finance and Management*, World Scientific Publishing, Singapore.
- Chernov, V., Dorokhov, O. and Malyaretz, L. (2012), "Construction of estimates in the choice of alternative solutions by using the fuzzy utilities", *Transport and Telecommunication*, 13(1), 11–17.
- Da Ruan (2000), *Fuzzy Sets and Fuzzy Information. Key Selected Papers by Lofti A. Zadeh*, Beijing Normal University Press, Beijing.
- Dorokhov, O. and Chernov V. (2011), "Application of the Fuzzy Decision Trees for the Tasks of Alternative Choices", *Transport and Telecommunication*, 11(2), 4–11.
- Efstathion, J. (2011), "Multi attribute decisions making using a fuzzy heuristic approach", *Intenn.J.Man –Machine studies*, 12(2), 141–156.
- Kravets, A. (1976), *Nature of Probability*, Idea-Press, Moskow. (in russian).
- Markowitz, H. (1959), *Portfolio Selection: Efficient Diversification of Investments*, Wiley, New York.
- Mcneil, M. and Ellen, T. (1994), *Fuzzy Logic, a Practical Approach*, Academic Press, London.
- Nedosekin, A. (2000), "Application of Fuzzy Set Theory to Problems of Financial Management", *Audit and Financial Analysis*, 2, 15–18. (in russian).
- Nowicki, E. (2001), *Problems of Strategic Management for Diversified Corporations*, Idea-Press, Moskow. (in russian).
- Ondruš, J. and Dicoová, J. (2013), "Potential of prediction quantification and trends in transport requirements as tool of transport management and development", *Transport and Telecommunication*, 14(4), 282–291.
- Wang, P., Da Ruan, Kerre E. (2007), *Fuzzy Logic A Spectrum of Theoretical & Practical Issues*. Springer, Berlin.