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The Comprehensive Method of Solving the Multiple Internal Rate of Return Problem

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ABSTRACT

The paper discusses the problem of the so called multiple internal rate of return with special emphasis of the author on the present attempts to solve the problem. Considering that none of the current numerous efforts to solve the problem of the multiple internal rate of return have provided a comprehensive solution to it, the author suggests a new approach and solution. The proposed and completely new integral method of calculating IRR and its redefining (as a rate that does not make the net present value equal to zero) enables each individual case, regardless of the character of the cash flow, to calculate the central rate and the corresponding central net present value. Central discount rate (C) represents IRR defined as the average rate of compounding of the investment with precondition of reinvestment. Besides, this method avoids the problem of the multiple IRR, while total (all) value of the area below the curve of the interdependent discount rates and net present value is more comprehensive and more real representative of the value of the NPV (Net Present Value) because the calculation takes into account all discount rates in the observed range. Finally, the method solves the problem of the appearance of the so called nonexistent IRR.

The mathematics is not to be considered as a drawback to the analysis, however, nor as method of reasoning which serious students can afford to neglect. Quite the contrary! The truth is that the mathematical method is a new tool of great power, whose use promises to lead to notable advances in Investment Analysis. Always it has been the rule in the history of science that the invention of new tools and new methods is the key to new discoveries, and we may expect the same rule to hold true in this branch of Economics as well.

J.B. Williams, The Theory of Investment Value, Preface ix, October, 1937.

INTRODUCTION

Internal Rate of Return (abbrev. IRR to be used herein) is one of the three most commonly used methods – indicators in the analysis of the economic and financial return of the investment. The term investment in this case equals to the definition by B. Graham (2006, p. 27) who says: "*An investment operation is one which, upon thorough analysis, promises safety of principal and an adequate return",* in his cardinal work *The Intelligent Investor*.

The basic definition of IRR is that it is a discount rate that equals Net Present Value (abbrev. NPV to be used herein) to zero, that is, that IRR is the highest discount rate that an investment, under the given circumstances, can bear, which indicates that it is also the average compounding rate (return) of the money invested in a certain investment project that will be gained under the precondition of the continuous reinvestment (Bendeković, 2007), (Pike, Neale, 1993), (www.investopedia.com/terms/irr.aspLiteratura). Precisely for the last aspect mentioned, IRR is one of the most favoured "banking methods" in calculating investment return. All advantages and disadvantages of this method are well known, one of the drawbacks being the so called paradox of the multiple IRR. Solving the problem (paradox) is the topic of this paper. Namely, the definition of IRR claims there is only one rate; however, there are cases when there are multiple internal rates of return or the so called paradox of the multiple IRR. The mathematical definition of IRR is:

$$0 = P_0 + P_1/(1+IRR) + P_2/(1+IRR)_2 + P_3/(1+IRR)_3 + ...P_n/(1+IRR)_n \quad (1)$$

Where: $P_0, P_1, \ldots$ P_n equals the cash flows in periods 1,2,.... n, respectively; and IRR equals the project's internal rate of return.

1. LITERATURE REVIEW

The attempts to solve the problem have existed in scientific circles for more than sixty years, more precisely, since 1955 when the problem was defined in the scientific paper *Three Problems in Rationing Capital* (Lorie and Savage, 1955). Most frequently highlighted challenge or problem related to IRR (Hazen, 2003) is the fact that cash flows can have contradictory internal rates of return. The awkward position of IRR lies in its usual interpretation where it is widely accepted that the multiple IRR presents significant and insoluble problem (White et al., 1998), that it is wrong (Canada et al., 1996; Sullivan et al., 2000), difficult to explain and complicated to interpret (Blank and Tarquin, 1989 White et al., 1998) or that it is completely wrong and useless (Bussey and Eschenbach, 1992). The core of this train of thought was given in the work of G. B. Hazen (2003) *„when multiple rates of return are found, there is no rational means for judging which of them is most appropriate for determining economic desirability"* (Thuesen and Fabrycky, 1989) that ends in the final thought *„Worse, when there are no real-valued internal rates of return, then the IRR approach must be abandoned altogether*" (Hazen, 2003). It is interesting that in the same work he emphasises that in case of the multiple IRR, their interpretation of return rate can be useful and valid only when using the problem approach as described in that work (Hazen, 2003) that can be classified as the so called engineering approach to the solution of the problem.

All numerous attempts to solve the above mentioned problem can be divided in three groups by the manner and approach to its solution: a) mathematical, b) engineering, and c) disregarding.

a) The first instance, the so called *mathematical approach* to the problem, is trying to solve, rather avoid, it so that the negative cash flow(s) are added to the positive ones and are discounted (Martic, 1980). The fundamental problem of this method is that the planned or realized cash flows that are result of business activities are actually fabricated or "tailored" so that the data given is not actual financial information from financial reports. This method conceals the cause of the appearance of the multiple IRR but does not solve it, and the same effect is obtained when suggesting to use the method of modified internal rate of return (MIRR) which represents the intervention in the current cash flow that is a result of the nature and dynamics of the planned (or existing) business activity.

b) The second, so called *engineering approach*, also "avoids" to a certain extent the problem of the multiple IRR since the method basically divides the course of planned investment in more stages aiming to avoid the changes of positive and negative cash flows, when distinctive stages are considered and evaluated individually, while in creating the cumulative cost effect and return of total investment they are taken altogether. Similarly to the previous approach/method, it leads to the conclusion that it is more about the avoidance or concealment of the cause of the problem rather than its solution (Hazen, 2003).

c) The so called *disregarding approach* to the problem of multiple IRR, most often found in textbooks, recommends to disregard the method of IRR as incalculable so that, accordingly, the method is not calculated (Van Horne, 1993; Berk and DeMarzo, 2014).

Chapter 2.1 provides more details about all the above mentioned attempts to solve the paradoxical problem of the multiple IRR. The basic assumption behind this paper or hypothesis is that there is a consistent solution to the problem in its mathematical, financial and economical aspects based on the use of infinitesimal calculus in calculating IRR. Besides the tools provided by the Microsoft Office, CurveExpert Professional 2.4 software was used for the purposes of this paper. The aim of the paper is to present coherently and comprehensively the solution to the problem of the multiple IRR that is methodologically consistent and can be utilized in each individual case, which, in effect, enables relevant use of this method in investment analysis and provides reliable calculation of IRR in cases when commonly used approaches lead an analyst to a certain epistemic ambivalence.

In the end of introductory remarks two more or less important things need to be said: a) this paper is based on the so called mathematical economics that is *de facto* deductive analysis and more theoretical than empirical in its nature, so that to this end A. C. Chiang (1994, p. 5) says: "*mathematical economics relates to the implementation of mathematics to purely theoretical views of economic analysis, with little or no care to such statistical problems as errors in measurements of variables relevant to research*"; b) considering this paper is the sole "product" of its author's thoughts on the problem, it will disregard the usual impersonal form and will use the first person singular instead. Besides, the new method of calculating IRR has proven to be better and more precise in calculating NPV, that is, the indicator of the cost effectiveness of the investment.

2. PROBLEM

The problem or the paradox of the appearance of the multiple IRR in the well known article *Three Problems in Rationing Capital* (Lorie and Savage, 1955) draws attention to the fact that cash flows can sometimes have more than one internal rate of return. The explanation of the problem is best illustrated in the example from the above mentioned article which shows the problem of the acquisition of the new oil pump that replaces the existing one. According to that investment proposal, the following cash flow can be established (Table 1).

Table 1. The planned cash flow for the investment in the acquisition of the new pump

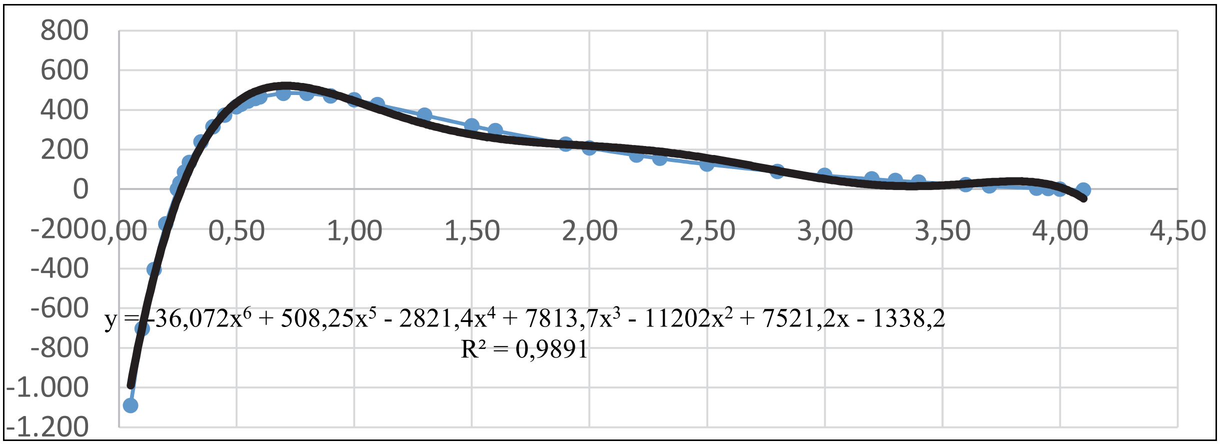
Year	0	1	2
Cash Flow $	-1.600	10.000	-10.000

Source: Lorie and Savage, 1955. pp. 229-239.

In the first instance, the new pump is more efficient from the existing one and, on the basis of the increase, the initial cost is followed by significant financial income as a direct result of the higher efficiency of the new one. However, the new pump consumes more oil which results in higher expenses and eventually the negative cash flow.

The problem arises in determining the internal rate of return for this investment since the result gives two values, IRR = 25% and IRR = 400%; the graphic representation of the relation between the discount rate and the net present value is given in the following graph (Graph 1) where the abscissa shows the discount rate and the ordinate NPV.

Graph 1. Graphic representation of the relation between NPV and discount rate of the investment

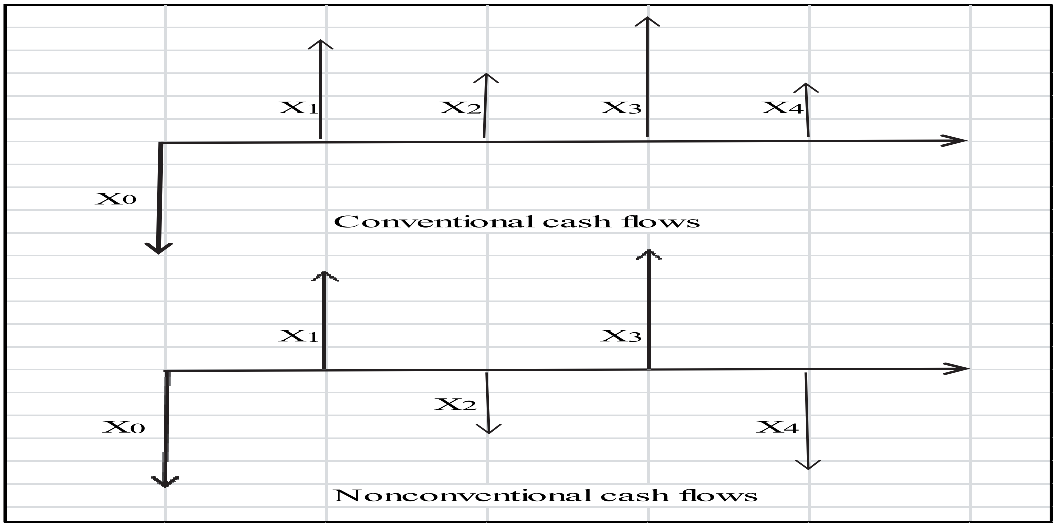


Source: the Author according to: Lorie and Savage, 1955, pp. 229-239.

The paradoxical situation is clearly presented in the above graph, the paradox being visible not only in the coexistence of the two internal rates but also in the illogical process of the function NPV(f)r as NPV for this investment in the discount rates from 0.00...1% to 25% shows negative values, while for the discount rates from 25% to 400% NPV gives positive values. Moreover, the NPV increases with the rise of discount rate in the interval between 25% and 69%, which is contrary to the very philosophy of discounting cash flows. Such or similar problem (paradox) appears in cases when there is a pattern of the so called non-conventional cash flow, for example, (-,+,+,+,+,-) or (-,+,+,-,+,-) or (-,-,-,+,+) and the like (Graph 2).

The essential condition for the appearance of the multiple IRR is the change of sign of the cash flows. However, additionally, the size and the relation of such changes in the cash flows are also important. Considering that computer programs make errors in calculating IRR even in cases of the multiple IRR as they show only one result, it is necessary to examine the relation of NPVs and different discounted rates in more detail in order to identify the possible existence of more than one.

Graph 2. Conventional and nonconventional cash flows



Source: the Author's calculation according to Moradi et al., 2012, p. 47.

In any case, one of the definitions of the problem of the appearance of multiple IRR can be: The multiple IRR problem occurs when at least one (or more) future cash inflows of a certain project is (are) followed by cash outflow(s), i.e. if the signs of cash flows change more than once, the project has non-conventional cash flow and the project can have (depending on cash flows value) more than one IRR.

2.1 Former attempts to solve the problem

2.1.1. Mathematical approach

As the introduction to the paper says, all former attempts of the solution to the problem can be divided in three groups and the so called mathematical approach is best illustrated in the example of a hypothetic investment (Martic, 1980, p. 35), as follows (Table 2).

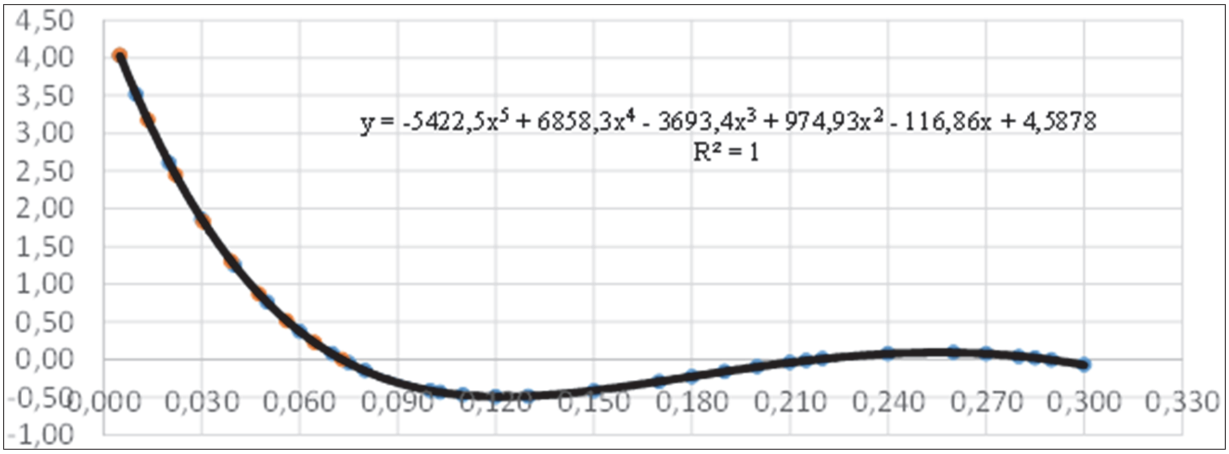
Table 2. Net cash flows by years

Year	0	1	2	3
Cash Flow $	-1.000,00	3.580,00	-4.260,00	1.684,80

Source: Martic, 1980, p. 35.

When calculating the internal rate of return, we get as many as three values, as shown in the next graph (Graph 3) where the abscissa shows the discount rate and the ordinate NPV.

Graph 3. Triple internal rate of return



Source: the Author's calculation according to Martić, 1980, 35

Mathematical method dictates to discount positive and negative results separately, then to add discounted negative ones to the invested amount (Martic, 1980, 37). The second, similar approach suggested is the discounting of the negative result for the previous year with the given discount rate and adding this value to the cash flow in the previous year. The described method achieves that all cash flows are positive (Martic, 1980, pp. 37-38). Both mathematical approaches, besides slightly complicated calculation detailed in the textbook by Lj. Martić, alter the original data and to a certain extent "falsify" the initial investment analysis, the problem being hidden rather than solved.

2.1.2. Engineering approach

This approach has similarities to the previous one; basically, it comes down to planned cash flows being divided into components or segments by using the so called *α-cut* method (Moradi et.al., 2012). The second solution this approach provides to the problem of the multiple IRR, which is also most recommended one, is to replace the indicator of IRR by the indicator MIRR (Modified Internal Rate of Return) or TRR (True Rate of Return) as modified, i.e. altered indicator of IRR based on the assumption of the continuous reinvestment of free cash flows of the project, while MIRR starts from the presumption of compounding by the rate of average expense of the capital of the project. Therefore, it is not possible to interpret MIRR as the average annual compound interest of the investment in the project. Besides, MIRR is similar to IRR but it is a completely different indicator and their replacement does not present the solution but avoidance of the problem.

$$MIRR = \sqrt[n]{\frac{\sum (CF)\cdot(1+rR)^{n-i}}{\sum Ki(1+r)^{-i}}} - 1 \qquad (2)$$

Where MIRR is modified IRR equal to nth root of the quotient of the sum of free cash flows during the duration of the investment compounded by the rate of the average capital cost and capital and sum of the discounted investment i.e. capital.

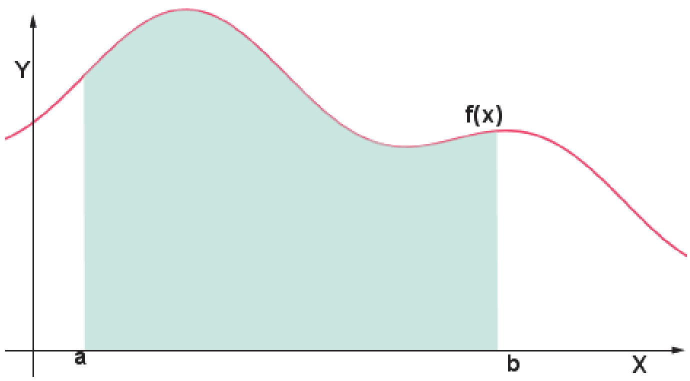
2.1.3 Disregarding approach.

A frequent solution to the problem (Cuthbertson and Nitzsche, 2008, p. 96) which *de facto* does not represent the attempt to solve the appearance of the multiple IRR but is a simple recommendation to omit IRR in such instances, and to establish the value of the cost effectiveness of the investment on the basis of other indicators, such as NPV (Van Horne, 1993, p. 165; Berk and DeMarzo, 2014, p. 212; Bendekovic et.al., 2007). It is interesting that many textbooks on finances and investments do not even mention the problem of the multiple IRR (Pike and Neale, 1993).

3. THE SOLUTION TO THE PROBLEM

The problem solution is based on the fact that each particular integral has a defined value that can be interpreted as a definite area under a curve f(x) (Chiang, 1984, 448), which is graphically presented as a shaded area in the following graph (Graph 4).

Graph 4. Definite integral as the area under the curve $\int_a^b f(x)dx$



Source: https://www.google.com/search?q=integrali+površina+ispod+krivulje+slike&tbm

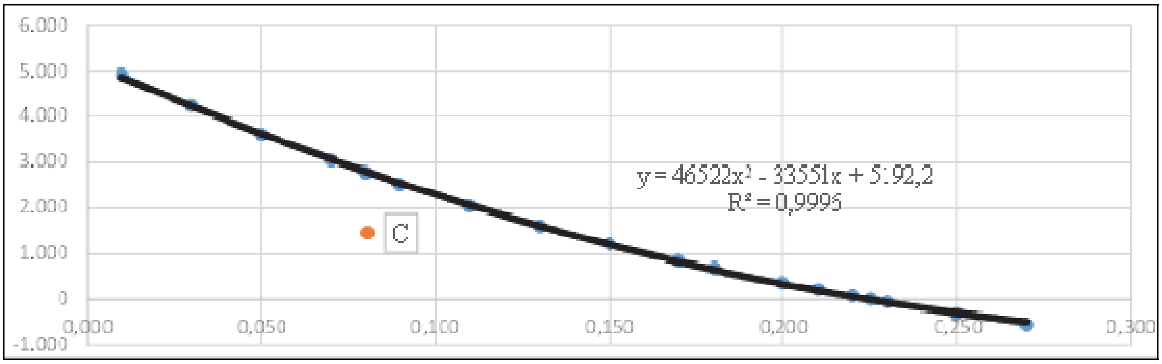
The following graph (Graph 5) presents the function, i.e. the relation of NPV and individual discount rates, where the abscissa shows the discount rate and the ordinate NPV, in case of the so called normal cash flows where only one IRR exists, which in the specific case amounts to 22% and where the NPV equals to zero. The curve consists of a series of points that represent the relation of the discount rate and the corresponding NPV. Regardless of the number of points taken into consideration, they are always certain points lying on the curve. Moreover, the more the points, the lesser the relative changes of the dependent variable, which makes them less significant.

However, if you integrate the above mentioned function in the interval from 0 (i.e. 0,0...1) to 0,22 on the axis, you get the value of the area (A) under the curve which means *all* points are taken into consideration, that is, all values of the relation between NPV and discount rate. Therefore, it is valid that:

$$A = \int_a^b f(x)\,dx \qquad (3)$$

Where: A is the area under the curve NPVf(r) bounded by a, and b, i.e. the range of discount rates.

Graph 5. Net present value as the function of discount rate



Source: the Author's calculation

The value A represents the total value of NPV for *all* discount rates in the observed range of values of discount rates. In case the area A>0, it means the investment, under the assumed conditions, is cost effective, however, in case A<0, the investment is not cost effective, while in the case of A=0 we can say it is a “bordering” case.

The next step in determining IRR is to define the centroid (C) in the given shape (area) bordered by the curve. According to the usual definition of the centroid, it is the point whose coordinates are the mean value of the coordinates of all the points of the given set. In its general form, the centroid (C), that is, the mean value is defined as *the point whose coordinates are the mean values of the coordinates of the points of the set*, as written in relation number 4.[1]

$$\bar{X} = \frac{\sum_{i=1}^{n} m_i X_i}{\sum_{i=1}^{n} m_i} \tag{4}$$

From the above mentioned it is visible that the centroid (C) is the average position of all points in the given shape, i.e. area. It is the point whose coordinates are the intersections of coordinates in that set, that is, it is the centre of the area. The previous definition leads to the conclusion that the centroid (C) equals to the average value of all relations in the given function NPVf(r). In other words, beside the discount rate defined by the centroid, we get the value of optimum NPV.

According to mathematical definition of the centroid it can easily be shown that the central point simultaneously represents the mean value of the function, that is, its average value as it is mathematically formulated in relation number 5.

$$\bar{p} = \frac{\sum_{i=1}^{n} A_i p_i}{\sum_{i=1}^{n} A_i} \tag{5}$$

The mean value of a function $Y=f(x)$, continued in interval (a, b) equals the sum of all values of the given function in that interval. In our case, the interval is (0, X), therefore, it is valid (Martic, 1987, p. 169):

$$\overline{f(x)} = \frac{\int_0^x f(t)dt}{x} \tag{6}$$

[1] ($\overline{X}$ = C)

In case we replace general signs by the sign for the average value of NPV, we get the adequate mathematical explication of the mean value of the function NPVf(r) as given in relation number 6.

$$\overline{NPV} = \frac{\sum_{i=1}^{n} A_i NPV_i}{\sum_{i=1}^{n} A_i} \tag{7}$$

3.1 The role of centroid (C) in solving the problem of multiple IRR

The previously described statement applied to cases when multiple IRR appears solves the problem in its entirety. In order to ascertain the statement, it will be demonstrated in two examples:

- Example A
- Example B

The hypothesis given in the paper introduction is that there is a consistent solution to the problem in its mathematical, financial and economical aspects based on integral calculus when calculating IRR.

The integral method consists of integrating function NPVf(r) (8) in the set interval and determining the area (A) under the curve by the numerical Newton-Raphson method of finding zero points (9) (https://www.saylor.org/site/wp-content/uploads/2011/11/ME205-3.2.1-TEXT.pdf) and then integrating the curve by Simpson method within intervals defined by zero points (10) https://math.dartmouth.edu/~m3cod/klbookLectures/406unit/trap.pdf which results in finding values of the area and the centroid of a shape under the curve (11) and (12),

$$NPV = f(r) \tag{8}$$

$$r_{i+1} = r_i + \frac{f(r_i)}{f'(r_i)} \tag{9}$$

$$\Delta r = \frac{b-a}{n} \tag{10}$$

$$\overline{NPV_r} = \frac{\Delta r}{3P} \sum SKr_n f(r_n) \tag{11}$$

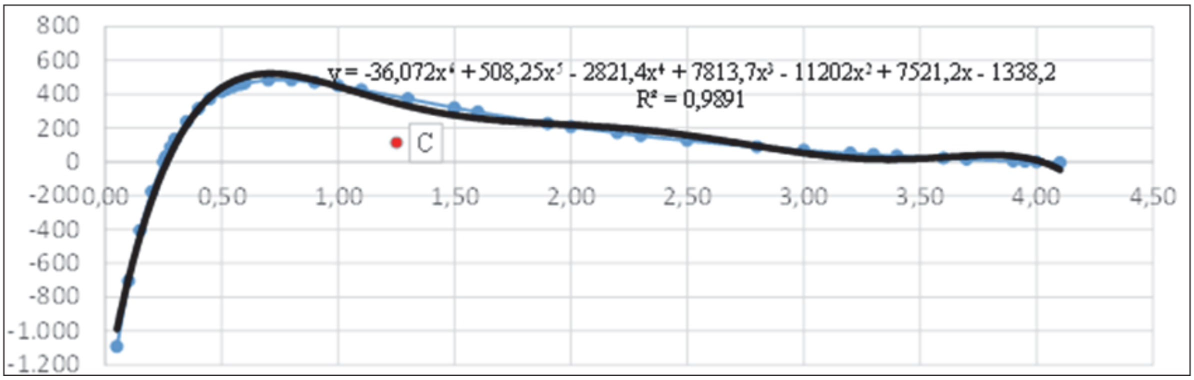
$$\overline{NPV_{NPV}} = \frac{\Delta r}{6P} \sum SKf(r_n)^2 \tag{12}$$

where *n* is the even number of segments bordered by the values of the function $f(r_n)$ and $f(r_{n+1})$, *SK* are the coefficients that alternate between 4 and 2, and NPV_r and NPV_{NPV} are the coordinates of the centroid on abscissa and ordinate.

Example A

Investment in the acquisition of the pump from the previously mentioned paper (Lorie, Savage, 1955, 229-239)

Graph 6. The representation of the relation between NPV and discount rate



Source: the Author's calculation

Table 3. The results of calculation

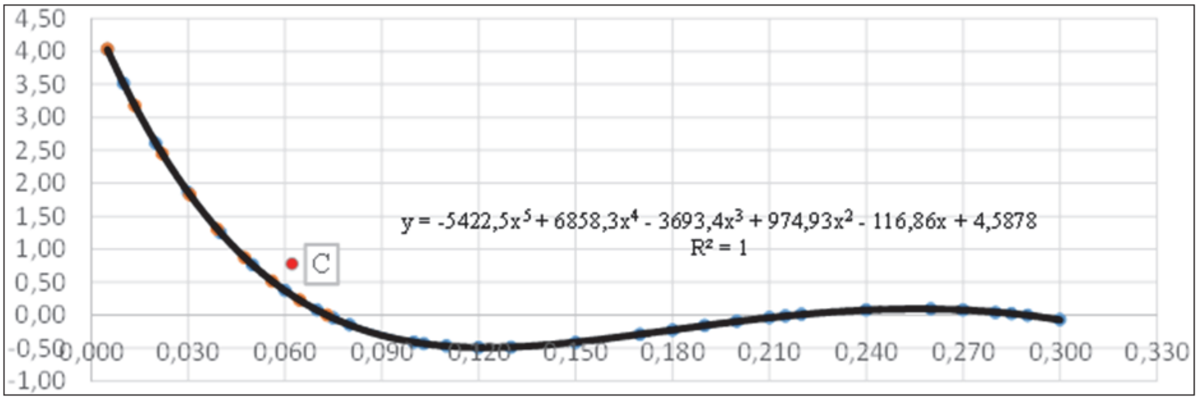
CALCULATION RESULTS		
$P = \frac{\Delta r}{3}\sum SKf(r_n)$	691	The sum of integrated area
$\sum \lvert P_i \rvert$	872	The sum of absolute values of individual segments
$\overline{NPV_r} = \frac{\Delta r}{3P}\sum SKr_n f(r_n)$	1,25	Centre of mass coordinate per r
$\overline{NPV_{NPV}} = \frac{\Delta r}{6P}\sum SKf(r_n)^2$	114	Centre of mass coordinate per NPV

Source: the Author's calculation

Example B

Hypothetic investment from the above mentioned example (Martic, 1980, p. 37).

Graph 7. The representation of the relation of NPV and discount rate



Source: the Author's calculation

Table 4. The results of calculation

CALCULATION RESULTS		
$P = \frac{\Delta r}{3}\sum SKf(r_n)$	0,06579	The sum of integrated area
$\sum \lvert P_i \rvert$	0,15247	The sum of absolute values of individual segments
$\overline{NPV_r} = \frac{\Delta r}{3P}\sum SKr_n f(r_n)$	0,0622	Centre of mass coordinate per r
$\overline{NPV_{NPV}} = \frac{\Delta r}{6P}\sum SKf(r_n)^2$	0,77529	Centre of mass coordinate per NPV

Source: the Author's calculation

3.2 The explanation of the obtained results

The previous two paradigmatic examples show the application of the methods of the solution to the problem or paradox of the multiple IRR. In contrast to former attempts to "solve" the problem, this paper presents the comprehensive method of solving the multiple IRR appearance as it is eliminated due to the completely different approach.

By calculating area (A) under the curve of function NPV dependent on discount rate, and in the next step calculating the centroid, the result is the mean value of the given function, that is, the central point Cr;NPV where *r*, that is, the discount rate represents IRR, that is, the average rate used to compound the invested amount under the precondition of reinvestment.

In this case, the IRR is not the discount rate where NPV = 0, but the average rate where the free cash flows of the investment are compounded and at that rate give a definite value of NPV. In both given examples, also valid generally, there are defined boundaries for calculating the area and the centroid starting from the value of the discount rate slightly bigger than zero (since compounding with zero rate is not compounding) to the last point of the intersection of the curve with abscissa, that can be calculated on the basis of the examination of the function flow, that is, bounding values defined by the range of discount rates that are meeting the defined criteria and that are logical.[2]

Example A. (Graph 6; Table 3) The usual method of calculating IRR in this example gives the dual solution IRR=25% and IRR= 400%. At first sight it can be said that it gives the negative result of investment because the negative result in the second year "annuls" the value in the first one, but for the nature of discounting the present value in the second year is lesser (i.e. less negative) that the positive value in the first year that is presented in the following table (Table 5).

Table 5. Discounted net cash flows

period	amount	discount rate	discount factor	discounted amount
1.	10.000	0,25	0,80000	8.000
2.	-10.000	0,25	0,64000	-6.400

Source: the Author's calculation

By applying higher discount rates the difference would be higher, and in longer period the paradox would be more visible because the higher the discount rate and the longer the period of discounting both positive and negative results, i.e. cash flows, would be lesser. Owing to the "paradox of discounting" in the given example there are two positive IRRs. However, by applying the suggested method, there is a logical value of NPV (NPV= 114) with central (Cr;NPV) discount rate (r=1,25). All discount rates in the observed range (0,03 – 4,00) result in total value of NPV (NPV= 691).

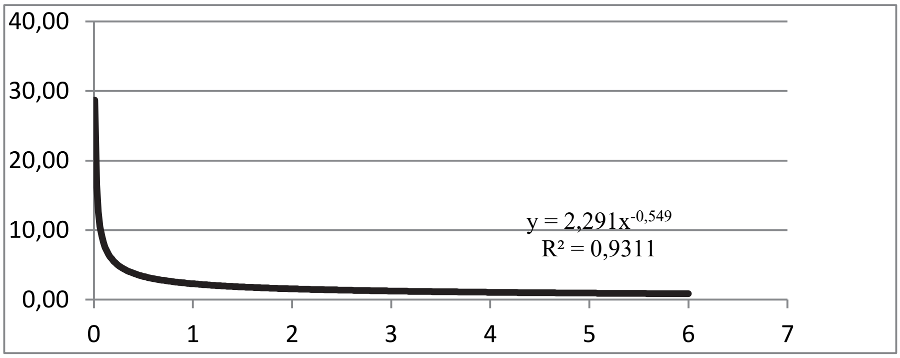
Example B. (Graph 7; Table 4) This case also presents the effect of the "paradox of discounting" but to a lesser degree than the previous one by applying the usual method of calculating IRR, which gives three rates: IRR= 0,073, IRR= 0,217 and IRR= 0,285, which presents the "insoluble" problem.

The application of the proposed method, i.e. process, in this paper and in this example demonstrates that the value of the area(s) in the range of discounted rates from 0,005 to 0,285 equals to 0,06579, and that the value of the centroid (C) on the coordinates is r=0,0622 and NPV=0,77529. The obtained result shows that the average rate of investment compounding for this investment amounts to 6,22% and that in the observed period and under the given conditions the average net present value of the investment will be 0,77529.

Furthermore, this model establishes the solution to the problem of the so called nonexistent IRR (Grinblat et al., 2002, pp. 348-350) which arises when the function of NPV and discount rate asymptotically approaches the abscissa as presented in the graph (Graph 8) where the abscissa shows the discount rate and the ordinate NPV.

[2] Let us analyse the present value (PV) of 1.000 € with discount rate of 700% in the fifth year. (PV=0,03€)

Graph 8. Nonexistent IRR



Source: the Author's calculation according to Grinblat et al., 2002, pp. 348-350.

CONCLUSION AND COMMENTS

Both presented cases apply the standard, that is, usual former method, i.e. process, in calculating IRR which produces confusing results so that the authors have been ignoring or “avoiding” this method or intervening more or less into data thus directly altering the nature of cash flows and obtaining acceptable (or unacceptable) results. Obviously, the main reason behind the paradox of the multiple IRR is the direction, dynamics and size of cash flows that besides the effect of the “paradox of discounting”, results in the above mentioned problem.

The completely new integral method of calculating IRR as well as it redefinition (as the rate that does not equal the Net Present Value to zero) given in this paper enables each individual case regardless of the character of the cash flows to calculate the central rate and the corresponding central net present value. Central discount rate (C) represents IRR defined as the average rate of the investment compounding with precondition of reinvestment. Furthermore, this method avoids the problem of IRR, and the total (whole) value of the area under the curve of related discount rates and net present value of the investment is more comprehensive and realistic representative of the value of the NPV because the calculation takes into account all discount rates in the observed range. Finally, this method eliminates, i.e. solves, the problem of (rare) appearance of nonexistent IRR.

Taking into consideration that I have theoretically defined the operationally acceptable solution to the problem of the multiple IRR, the aim of the paper and the stated hypothesis have been achieved.

REFERENCES

Bendekovic, J. et.al. (2007), *Preparation and rating of investment projects*, FOIP-1974, Zagreb (in Croatian).

Berk, J., DeMarzo, P. (2014), *Corporate Finance*, Pearson Education Inc., Boston, Mass. (3rd Edition).

Blank, L. T., Tarquin, A. J. (1989), *Engineering Economy*, McGraw–Hill, New York.

Bussey, L. E., Eschenbacht, T, (1992), *The Economic Analysis of Industrial Projects*, Prentice Hall, U.K.

Canada, J. R., Sullivan, W. G., WHITE, J. A, (1996), *Capital Investment Analysis for Engineering and Management*, Prentice–Hall, Upper Saddle River, New Jersey.

Chiang, A. C. (1984), *Fundamental Methods of Mathematical Economics*, McGrow–Hill, International (3rd Edition).

Cuthberson, K., Nitzche, D. (2008), *Investments* (2nd Edition), J. Wiley & Sons, Ltd. Chichester West Sussex, England.

Graham, B. (2006), Intelligent Investor, Masmedia, Zagreb (in Croatian).

Grinblatt, M., Titman, S. (2002), *Financial Markets and Corporate Strategy* (2nd Edition), The McGrow-Hill Companies, USA.

Hazen, G. B. (2003), “A New Perspective on Multiple Internal Rates of Return”, *The Engineering Economist*, Vol. 48, No. 1, pp. 31-51.

Lorie, J. H., Savage, L.J. (1955), “Three Problems in Rationing Capital, The Journal of Business”, Vol. 28, No. 4, pp. 229-239.

Martic, Lj. (1987), *Mathematical methods for economic analysis I*, Narodne novine, Zagreb (in Croatian).

Martic, Lj. (1980), *Quantitative methods for financial and accounting analysis*, Informator, Zagreb (in Croatian).

Moradi, B., Shakeri, H., Zangeneh, S. N. (2012), “Solving the Paradox of Multiple IRR’s in Engineering Economic Problems by Choosing an Optimal α-cut”, *International Journal of Industrial Engineering & Production Research*, Vol. 23, No. 1, pp. 45-52.

Pike, R., Neale, B. (1993), *Corporate Finance and Investment – Decisions & Strategies*, Prentice Hall International (UK) Ltd.

Sullivan, W. G., Bontadelli, J. A., Wicks, E. M., Degarmo, E. P. (2000), *Engineering Economy*, Prentice–Hall, Upper Saddle River, New Jersey, USA

Thuesen G.J., Fabricky, W.J.,(1989) Engineering Economy, Prentice–Hall, Englewood Cliffs, New Jersey, USA.

Van Horne, J. C, (1993), *Financial Management and Policy*, 9th Edition, Mate d.o.o., Zagreb (in Croatian).

White, J. A., Case, K. E., Pratt, D. B., Agee, M. H. (1998), *Principles of Engineering Economic Analysis*, Wiley, New York, USA.

Williams, J. B. (1938), *The Theory of Investment Value*, Harvard University Press, Cambridge, Mass. USA (Reprint of 1st Edition 2014).

Wells, D. (1991), The Penguin Dictionary of Curious and Interesting Geometry, Penguin, London.

Internet pages:

http://mathworld.wolfram.com/GeometricCentroid.html (accessed on 18 May 2017)

www.investopedia.com/terms/irr.aspLiteratura (accessed on 28 June 2017)

https://www.saylor.org/site/wp-content/uploads/2011/11/ME205-3.2.1-TEXT.pdf (accessed on 8 May 2017)

http://mathforcollege.com/nm/mws/gen/07int/mws_gen_int_txt_simpson3by8.pdf (accessed on 19 February 2017)

https://www.google.com/search?q=integrali+površina+ispod+krivulje+slike&tbm (accessed on 22 August 2018