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Modeling and Visualization of Quadratic Assignment Problems on the Example of Plant Location

SLAVOMIR VUKMIROVIC [1], DRAGO PUPAVAC [2], and JUSUF SEHANOVIC [3]

[1] University of Rijeka, Faculty of Economics Rijeka, Rijeka, Croatia
[2] Polytechnic of Rijeka, Rijeka, Croatia, e-mail: drago.pupavac@veleri.hr
[3] Juraj Dobrila University of Pula, Department of Economics and Tourism «Dr. Mijo Mirkovic», Pula, Croatia

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ABSTRACT

The object of our research is the visualization of use and functioning of modelling programming languages used in solving complex quadratic assignment problems. In accordance with our research object, we have established a hypothesis statement that the entire process of creating a model for solving the quadratic assignment problem can be visualized using modelling programming languages in Excel interface. This means that programming languages can be easily used by economists in quantitative modelling and solving of complex, optimisation problems the easy way.

INTRODUCTION

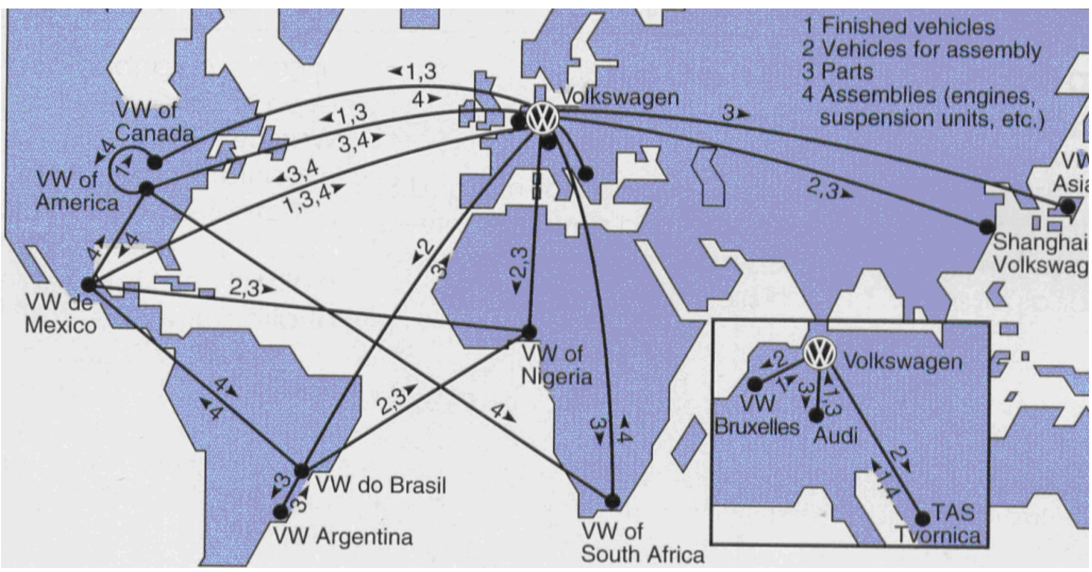
One of the most important strategic decisions made by many companies is where to locate their operations. The international aspect of these decisions is a consequence of the globalization process. Managers throughout the world are using different methods for solving location problems because location greatly affects both fixed and variable costs (Streimikiene, Mikalauskiene and Barakauskaite-Jakubauskiene, 2011; Kaplikski and Tupenaite, 2011; Ciegis, Dilius and Mikalauskiene, 2015). Location may determine up to 10% of the total cost of an industrial firm (Heizer and Render, 2004). Once management is committed to a specific location, many costs are firmly in place and difficult to reduce. Consequently, scientific approach to determine an optimal plant location can make the difference between success and failure. The location decision often depends on the type of business. For industrial location decisions, the strategy is usually minimizing costs. Accordingly, in this work we discuss the possibility of determining the optimal allocation of a set of *n* plants to a set of *n* locations on the global market. The objective is to select the optimal combination of pairs of plant location assignments with the goal of minimizing distances and flows (material, spare parts, semi finished products, and components) between the plants. Such problems can be solved using a mathematical model for the problem of quadratic assignment (QAP – Quadratic Assignment Problem). The QAP formulation requires an equal number of distribution centres and locations. If the number of locations is greater than the number of distribution centres, a fictitious location (dummy variable) should be created and a zero flow between each of these and distribution centres must be assigned. If the number of distribution centres is greater than the number of locations, problem cannot be solved.

After the introduction part, in the second part of the paper we describe and discuss the quadratic assignment problem of plant location through the example of Volkswagen Group logistics networks. In the third part of the paper, an illustrative example of QAP visualization is given, while in the forth part we discuss a process of creating a mathematical model to solve a quadratic assignment problem through the example of plat location problem. In the fifth part we describe and elaborate a process of model-based visualization to solve a QAP of plant location using synergy between LINGO (programming language) to object-oriented programming language (VBA for Excel). In the final part of the paper, we synthesize our research findings.

1. DESCRIPTION AND SIGNIFICANCE OF QUADRATIC ASSIGNMENT FOR PLANT LOCATION PROBLEM

The question “Where should we locate?” is more prominent in the minds of executives than it has ever been. Over the past three decades, business activities have become increasingly mobile. Many activities in the value chain, however, are tied neither to customers nor to resources but instead are mobile. They can be placed in any of numerous locations and moved as circumstances change. Mobile activities gravitate toward the location that allows them to be performed the most effectively at the lowest total cost. Today improvements in information, communication, and logistics technology allow firms to serve many markets from a distance, spread discrete activities around the globe, and coordinate them in a global system. Thus, managers must increasingly decide not only which countries to serve but also where to locate each activity in the value chain (Porter and Rivkin, 2012). The logistics networks of the Volkswagen Group, one of the world’s leading companies, is a typical example of the global value chain (cf. Map 1).

Map 1: Global logistics networks of the Volkswagen Group



Source: Heizer and Render, 2004, p. 311.

In the Map 1, we see that VW Mexico ships unassembled vehicle sets and spare parts to VW Nigeria, and components such as engines, and shock absorbers etc. to VW Brazil, while concurrently receiving spare parts and components from Germany. The range of location options such production has expanded dramatically as many countries have stabilized their macroeconomic policies, opened their markets, improved their infrastructure, strengthened their economic institutions, and upgraded the skills of their workforces. Countries that used to attract activities only on the basis of natural resources or cheap labour can now vie for activities that rely on more skill and involve more-complex manufacturing or services. Linear programming is the most commonly used method of solving such problems.

Two following aspects should be considered when defining spatial logistics chain configuration: 1) Level of centralization or geographic concentration of functions, and 2) Level of coordination of dispersed activities. A producer can either have all its business activities placed in a specific location wherefrom goods or services are being supplied onto the whole world market, or can have its plants all over the world from which goods or services are being supplied onto each individual local market. Concentrated production means scale effects, use of highly specialized knowledge and skills, and possibility of controlling the performance all over the world. Dispersive configuration of logistics chain allows for more attentive monitoring of market developments, greater ability of adapting to market demands and consumer needs. Dispersion of activity in logistics chain ensures greater strategic flexibility, and allows producers to provide resources from more convenient locations, decrease stock, have less warehouses, and offer faster delivery. Accordingly, the following three major production models within global logistics chains can be identified:

I. Centralized production model – Production activity is concentrated in a single or maximum two locations covering the entire world market. This model has positive impact on economies which are likely to achieve significant scale effects;

II. Interrelated, dispersed production plants – Production, plants and assembly plants are placed at interrelated, centrally coordinated multiple, different locations. This is the most favourable model for economies where the best solution seems to be to have individual components manufactured in different countries, or to purchase individual components from multiple, different countries. In such a model, plants where finished products are assembled are usually located near large markets, but components are purchased from widely dispersed locations.

III. Decentralized, dispersed production – This model is characterized by a larger number of locations all over the world but only partly interrelated. This is often seen in economies which do not exhibit the scale effect. This model is also suitable for economies characterized by significant differences between factor cost and technology from one market to another. Many factors (wage levels, skills availability, utility rates, taxes, subsides, shipping costs and reliability, local productivity, supervision costs, etc) affect the profitability of operating in a certain locale. Locating factors are complex, interrelated and dynamic. Locating an activity in one country often has ripple effects on activities elsewhere. Because many managers are still trying to find the appropriate way to improve processes for making location decisions. Four major methods are used for solving location problems: the factor-rating method, locational break-even analysis, the center-of-gravity method, and the transportation model (Heizer and Render, 2004).

2. ILLUSTRATIVE EXAMPLE OF VISUALIZATION OF QAP MODEL

Quadratic Assignment Problem (QAP), as the name suggests, is a typical assignment problem where the goal function is a quadratic function or the second-order polynomial, which means that it includes the product of the two variables (Burkard et al. 1998). The assumption is that a company wants to minimize material flows between its plants by determining the optimal combination of pairs of plant-location assignments. Flows of materials moving between factories and distances between locations where plants can be allocated have been defined. Scheme 1 shows distances between locations, and scheme 2 reveals flows of materials between plants. Flows of materials have been expressed in average number of containers being transported per day.

Scheme 1. Matrix of distances between locations

L11	L12	L13	
0	10	5	L13
	0	13	L23
		0	L33

Scheme 2. Matrix of flows between plants

P11	P12	P13	
0	42	90	P13
	0	110	P23
		0	P33

Scheme 1 indicates distances between locations, and scheme 2 reveals quantities of materials being transported between plants. *L* means location, while *P* means plant. Each pair of indexes next to location mark means distance between the two locations. We can see in the scheme 1, for instance, that the distance L12 between the location 1 and location 2 is 10, while L13 indicates that the distance between the location 1 and location 3 is 5 etc. Data in the scheme 2 have been marked in the same way. In the Table 1, we see that N=3 (3x3 assignment), which means that it is possible to make 9 combinations of pairs of plant location assignments from which we can select that with the minimum product of distances between locations and quantities of materials being transported between those.

Table 1. Possible combinations of sumproducts of the distances and flows

	Locations	Plants	
L1	10	42	P1
L2	13	110	P2
L3	5	90	P3

The initial model can be built in the same way as the model shown in the scheme 3, graphically indicating the model goal. It is necessary to calculate the optimal combination of assignments for each location L_i and plant P_j, so that each location is assigned exactly one plant and each plant is assigned exactly one location with the goal of minimizing the transportation cost.

Scheme 3. Initial QAP model

L1	L2
L3	

P1	P2
P3	

Li, Pj	Li, Pj
Li, Pj	i=1,2,3

Scheme 4. QAP model solution

L1	L2
L3	

P1	P2
P3	

L1, P2	L2, P3
L3, P1	i=1,2,3

The solution reveals the value of minimum cost calculated using a model for optimizing the assignment of plants to locations. The optimal solution in the scheme 4 indicates that the plant 1 has been assigned to the location 2, plant 2 has been assigned to the location 3, and plant 3 has been assigned to the location 1.

We have selected the optimal combination of products, and included those into the equation, to determine the minimum cost, respecting the given limitations. The sum of included products gives the minimum result. We can see in the table that replacing any product that has been included into the equation with another product of values of pairs in the columns that has not been included into the equation is not likely to produce a more favourable result or lower value than the one computed. This has been expressed in terms of the following mathematical formula: L12*P13+L23*P12+L13*P23 = 10*90+13*42+5*110 = 1996.

The table shows optimal combination of products of distances and flows whose sum is the minimum cost. Distances and flows have been shown in the columns C and D. It is potentially possible to include each pair of products of the values in the columns when approaching the problem, so that a single value from one column can be multiplied with any other value from another column. This means that the number of limitations in quadratic programming would be equal to the second power of the number of rows as opposed to single dimensional limitations where the number of inequations equals the number of rows.

In the scheme 5 we have compared the result of optimal solution with a result which could have been obtained by a random, unintentional, or unplanned assignment where production plants, for instance, could be assigned to locations according to their ordinal numbers. In other words, the plant 1 could have been assigned to the location 1 etc. The comparison of results revealed difference between the costs that accounts for a decrease in cost achieved by using modelling languages and QAP optimization software.

Scheme 5. Comparison of results

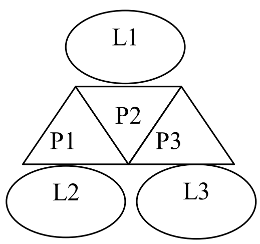
L1, P2	L2, P3
L3, P1	**R1=1996**

L1, P1	L2, P2
L3, P3	**R2=2300**

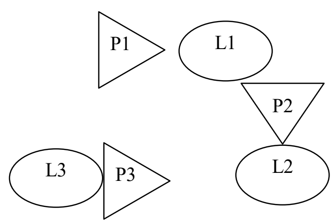
Saving = R1-R2
= 304

Optimal assignment of plants to locations can also be shown graphically (cf. scheme 6). Scheme 6a represents a random assignment giving a cost of 2300 units of money; Scheme 6b indicates optimal plant location assignment with the minimum cost of 1996 units of money. It follows that optimal plant location assignment ensures cost almost 15.2 % lower.

Scheme 6a



Scheme 6b



We are able to compute cost and define pairs to be assigned to each other by adding the products of distances and flows for all pairs of plant location assignments respecting the given limitations. The minimum cost formula can easily be checked by replacing any product multiplied with 1 in the total sum of products with product multiplied with 0; product replacement will not produce more favourable results. Or, in other words, the value of minimum cost obtained in such a way will never be lower.

3. MATHEMATICAL MODEL OF QUADRATIC ASSIGNMENT PROBLEMS FOR PLANT LOCATIONS

The Quadratic Assignment Problem was originally introduced in 1957 by Koopmans and Beckman who were trying to model a plant location problem. The problem models the following real-life problem: There are a set of *n* plants and a set of *n* locations. For each pair of locations, a distance is specified and for each pair of plants a weight or flow is specified (e.g., the amount of supplies transported between the two plants). The problem is to assign all plants to different locations with the goal of minimizing the sum of the distances multiplied by the corresponding flows. Intuitively, the cost function encourages factories with high flows between each other to be placed close together (Burkard et al. 1998; Sunderesh and Heragu, 1997). We have demonstrated the mathematical model used to solve a QAP problem by an example of factory location assignment. Table 2 illustrates an example of problem of allocating factories to locations with the goal of minimizing the cost of material transportation between the factories.

Table 2. Distances between locations and unit transportation costs

Table 2a. Material flow cost

	A	B	C	D	E
12	**Distances betwen locations (D)**				
13	**R S**	**2**	**3**	**4**	**5**
14	**1**	100	80	70	40
15	**2**		80	100	50
16	**3**			70	90
17	**4**				40

Table 2b. Distances between locations

	A	B	C	D	E
2	**Material flow costs (M)**				
3	**R S**	**2**	**3**	**4**	**5**
4	**1**	2	10	10	4
5	**2**		15	30	35
6	**3**			20	40
7	**4**				10

The goal is to allocate factories to the locations in an optimal way to achieve minimum product of cost for the quantities of materials being transported between the production plants and distances between their locations. In the example, distance between the location d_j and location d_k is marked as d_{jk}, and material being transported between the plant m_i and plant m_l along those distances is marked as m_{il} Plant m_i being assigned to location d_j is marked as x_{ij}, given that x_{ij} = 1. This has been expressed in terms of a mathematical formula below (Burkard et al. 1998; Sunderesh and Heragu, 1997):

$$\text{Min} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} \sum_{l=1}^{n} x_{ij} x_{kl} m_{il} d_{jk}$$

The model reveals that there is a set of N factories and a set of N locations which are to produce N*N variables of type x_{ij} and N*N*N*N variables of type z_{ijkl}. The formula reveals that the size of model is exponentially increasing by accretion in the variable N. It follows that models with greater values N require specialized programs with robust computing platforms.

Smaller size QAP models can be converted into Integer Linear Programming so that the product $x_{ij}x_{kl}$ is being replaced with a single variable z_{ijkl}. In this case, the goal function can be expressed in terms of a mathematical formula below (Sunderesh and Heragu, 1997):

$$\text{Min} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} \sum_{l=1}^{n} z_{ijkl} m_{il} d_{jk}$$

If the matrices are symmetric, data can be written as shown in the tables 2[a] and 2b. The goal function has been expressed in terms of a mathematical formula below:

$$Min \sum_{i=1}^{n} \sum_{j=1, j<k}^{n} \sum_{k=1}^{n} \sum_{l=1}^{n} z_{ijkl} m_{il} d_{jk} + \sum_{i=1}^{n} \sum_{j=1, j>k}^{n} \sum_{k=1}^{n} \sum_{l=1}^{n} z_{ijkl} m_{il} d_{kj}$$

We need to define limitations as well. Limitations exist to ensure that each factory is placed onto exactly one location. In other words, each location can be assigned exactly one factory. These limitations can be defined in terms of the formulas (1) and (2) below.

Each factory l can be assigned exactly one location k.

$$\sum_{l=1}^{n} x_{lk} = 1 \qquad for\ k = 1,..,n \qquad (1)$$

Each location k can be assigned exactly one factory l.

$$\sum_{k=1}^{n} x_{lk} = 1 \qquad for\ l = 1,..,n \qquad (2)$$

The variable x is binary and has been expressed in terms of a mathematical formula below:

$$x_{ij} \in \{0,1\} \qquad for\ i, j = 1,..,n \qquad (3)$$

The following limitations apply as well. If the object l has been assigned to the location k, than there should be a location j that is different from k (j#k) for each object i that is different from l (i#l), to which the object i can be assigned. This limitation has been expressed in terms of the following mathematical formula (4):

$$x_{lk} = \sum_{j=1, j\#k}^{n} (z_{ijlk} + z_{lkij}) \quad for\ i\#l \qquad (4)$$

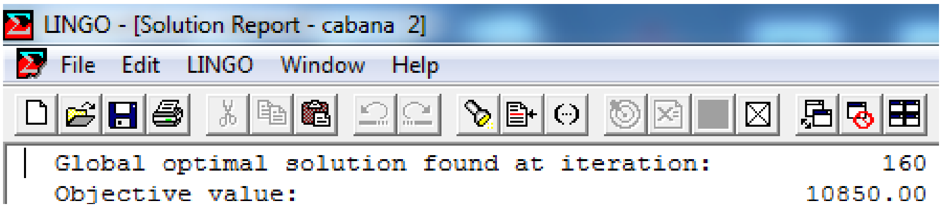
If the object l has been assigned to the location k, than there should be an object i that is different from l (i#k) for each location j that is different from k (j#k), to which location j can be assigned. This limitation has been expressed in terms of the following mathematical formula (5).

$$x_{lk} = \sum_{i=1, i\#l}^{n} (z_{ijlk} + z_{lkij}) \quad for\ j\#k \qquad (5)$$

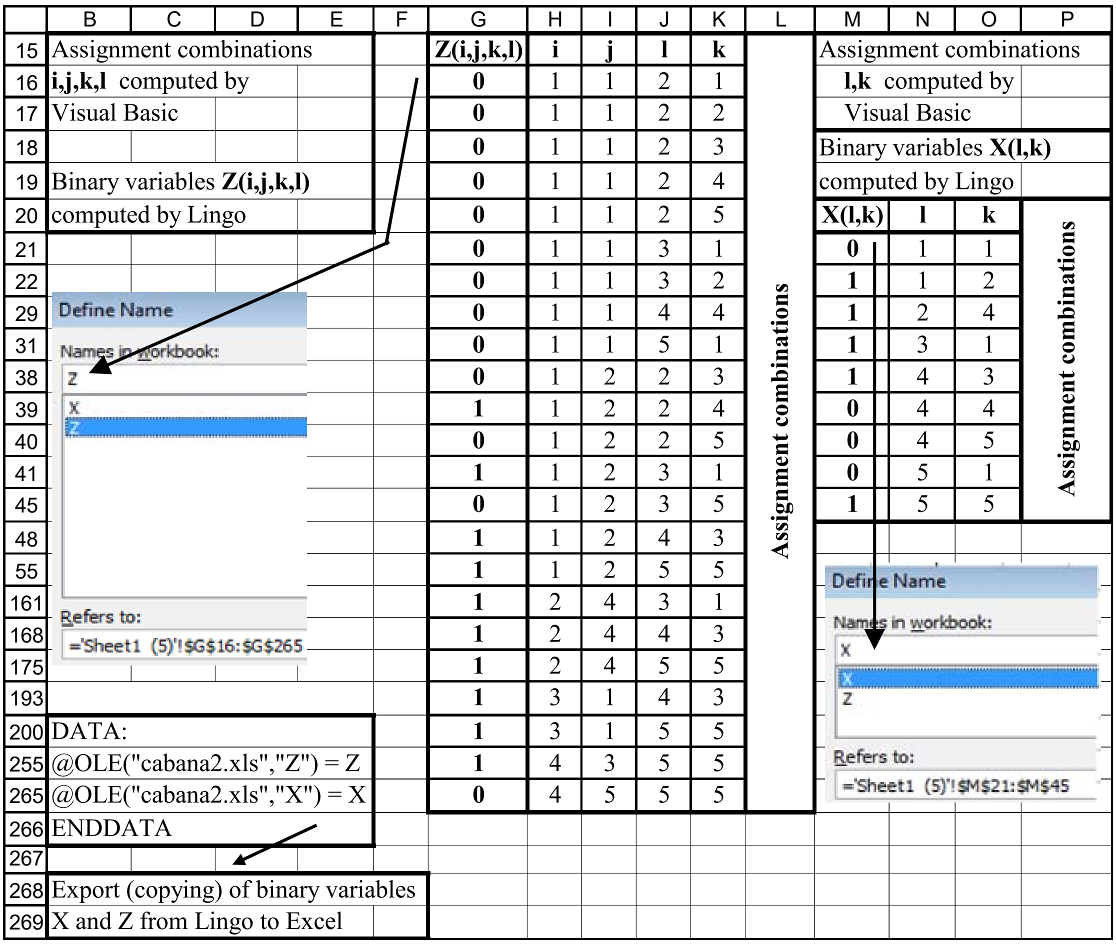
4. VISUAL MODEL DESIGN OF THE QUADRATIC ASSIGNMENT PROBLEM FOR PLANT LOCATION PROBLEM

Mathematical models can be created using programming languages for mathematical modeling. In our example we are going to use LINGO's modeling language. Modeling languages enable us to structure a problem in a mathematical form of indexes and bases. The main characteristic of a modelling language is the ability to group similar entities into sets. Once the entities have been grouped as a set, they are represented by the characteristics of that set. In such a way, entity groups can be represented using a single algebraic formulation. LINGO software is a program designed to effectively solve problems in mathematical programming (Walkenbach, 2003). Scheme 7 shows how an optimal value is computed using LINGO software.

Scheme 7. Computation of optimal solution using LINGO software



Scheme 8. Model of computation for binary variables and optimal combinations of plant location assignments



Scheme 8 shows the model for computing values of binary variables and optimal assignment combinations. We have created a graphic representation of the QAP problem using a synergy between LINGO's modelling language to an object-oriented programming model, VBA for Excel created by the authors of this work. We have defined the cells in the Excel table in form of vectors for the variables X and Z with the command *Define Name* (Walkenbach, 2003). We see in the Scheme 8 that the variable X has been assigned the cell range MP21:M45, and variable Z has been assigned the cell range G16:G265. We are able to copy the values from the LINGO program into the Excel table with the command *Object Linking and Embedding* (OLE), as shown in the scheme (Schrage, 2003). The values of binary variables X and Z computed in the LINGO program have been copied into the Excel table in such a way. The cell range corresponds to the number of possible combinations. The variable Z indicates possible combinations of the products of values of cost of the material flows (Table 2a) and distances between locations (Table 2b). There are 250 such combinations. The variable X indicates possible combinations of assigning factories to locations and there are 25 of those. For easy reference, the rows where binary values equal 0 have been hidden.

We can see in the scheme 8 that optimal combinations have been assigned value 1. Optimal combinations of plant location assignments have been defined in the cell range M21:O45 having a variable value of X(l,k) = 1. It can be seen in the table that optimal combinations of plant location assignments are ({1,2], {2,4}, {3,1}, {4,3} and {5,5}. Cell range G16:K265 contains optimal combinations of the values of products in the matrices of material cost and distances between locations. Optimal combinations of plant location assignments have been defined by the binary variable Z(i,j,l,k) given that Z(i,j,l,k) = 1.

Scheme 8 is the visualization of limitations set up in our mathematical model for solving the quadratic assignment problem. Cell range M21:O45 reveals that each factory l has been assigned exactly one location **k** that corresponds to the limitation (1) in the mathematical model, and each location **k** has been assigned exactly one factory l that corresponds to the limitation (5). Cell range G16:K265 reveals that for each combination of the value l (representing factory l) and value **k** (representing location k) there is exactly one value **i** (representing factory i), which is different from the value l and to which each location **k** being different from the value **j** can be assigned. In this way we are able to visualize the limitation (4) in the mathematical model. Similarly, we are able to visualize the limitation (5).

Scheme 9. Visual model for QAP solution in Excel interface

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1							M(r,s)*D(r,s)	Z(i,j,l,k) * (		M(i,l)	*	D(j,k) +		T(i,j,l,k)	
2	**Material flow costs (M)**								+	M(i,l)	*	D(k,j)) =			
3	**R S**	**2**	**3**	**4**	**5**		(1,2)*(2,4)	**Optimal assignment**	**1**	**Material flow costs**	**2**	**Distances between locations**	**100**	**Minimal costs**	**200**
4	**1**	2	10	10	4		(1,3)*(1,2)		**1**		**10**		**100**		**1000**
5	**2**		15	30	35		(1,4)*(2,3)		**1**		**10**		**80**		**800**
6	**3**			20	40		(1,5)*(2,5)		**1**		**4**		**50**		**200**
7	**4**				10		(2,3)*(1,4)		**1**		**15**		**70**		**1050**
8							(2,4)*(3,4)		**1**		**30**		**70**		**2100**
9	R - row index						(2,5)*(4,5)		**1**		**35**		**40**		**1400**
10	S - column indeks						(3,4)*(1,3)		**1**		**20**		**80**		**1600**
11							(3,5)*(1,5)		**1**		**40**		**40**		**1600**
12	**Distances betwen locations (D)**						(4,5)*(3,5)		**1**		**10**		**90**		**900**
13	**R S**	**2**	**3**	**4**	**5**						**Total costs**			=	**10850**
14	**1**	100	80	70	40										
15	**2**		80	100	50		**Z(i,j,l,k)**	**i**	**j**	**l**	**k**		**X(l.k)**	**l**	**k**
16	**3**			70	90		1	1	2	2	4		1	1	2
17	**4**				40		1	1	2	3	1		1	2	4
18							1	1	2	4	3		1	3	1
19	**Optimal plants / locations**						1	1	2	5	5		1	4	3
20	**assignment**						1	2	4	3	1		1	5	5
21	**Plant**		**Location**				1	2	4	4	3				
22	**1**		**2**				1	2	4	5	5				
23	**2**		**4**				1	3	1	4	3				
24	**3**		**1**				1	3	1	5	5				
25	**4**		**3**				1	4	3	5	5				
26	**5**		**5**												

Scheme 9 shows the visual model for QAP solution in Excel interface. Combinations of plant location assignments have been sorted based on the values of binary variables followed by selecting only optimal combinations as shown in the scheme 9. For instance, the cell range M21:O45 in the scheme 8 contains optimal combinations sorted based on the binary variable values X(l,k). The cell range M16:M20 in the scheme 9 contains selected optimal combinations of factory l to location k assignments. In a similar way, we have sorted possible combinations and selected optimal combinations of values of the products of material cost M(i,l) and distances between locations D(j,k).

Cell range G1:O13 contains minimum cost computed in the QAP model. The values of variable Z(i,j,l,k) equalling 1 indicate optimal combinations of plant location assignments. The column G contains indexed cells in the matrices of material cost and distances between locations that correspond to optimal combinations in the cell range G16:K25.

For instance, combination of values in the range G16:K25 for the factories i (1) and l (2) in the first row indicates position of the first row and second column in the material cost matrix revealing that the unit value of cost is 2. We can also see in the first row of the table that combination of values for the locations j (2) and k (4) indicates position of the second row and forth column in the distance matrix which reveals that the distance is 100. Column T(i,j,l,k) in the cell range D3 contains the product of these two values equalling to 200. The remaining values of products of optimal combinations of material cost and distances between locations have been computed in the same way.

Cell range D14 contains the sum of products which corresponds to the optimal value of minimum cost equalling to 10850. The obtained value of total minimum cost equals the value computed using LINGO software.

CONCLUSION

Graphic representation of how the input data, programs as processes and output data are structured enables us to understand the entire process of creating models for solving the quadratic assignment problems using information technology, and how the mathematical model is working. This allows managers to use models flexibly regardless of their size (number of variables), directions of flows of goods or other flows (one direction or multiple direction), or kinds of problems. For instance, the model illustrated in this paper can be used for completely different kinds of problems which are solved using quadratic assignment method. Examples of such problems include the minimizing of flows of people and documents between the offices in a building, or minimizing the flows of material between the installations in a plant and the like.

A manager who is able to understand how a computing model for solving the quadratic assignment problems is created and is working will know how to manage purchasing processes and use of equipment needed to implement quadratic assignment problem applications. In addition, it is important to know to decide hardware capacity with respect to exponential accretion in variables, after adding each new vector in a quadratic matrix. For instance, a student version of LINGO software will work at 8x8 matrix level; after that you will need to purchase a professional version of the software. Problems that occur in the real-life require matrices of larger size, which calls for an evaluation of maximum matrix size. In other words, it is important to evaluate the level of profitable investing in new hardware.

In this work, we have proven the hypothesis that methodological approach to visualization and systematic use of modelling programming languages in Excel interface enables us to visualize the entire process of creating models for solving the quadratic assignment problems. We have proven our hypothesis through an example of using LINGO's modelling language to solve a problem of quadratic assignment of distribution centres to locations. We have compared the result obtained using the QAP method (10850) with the result computed by a random, unintentional, or unplanned assignment (12000) where, for instance, distribution centres could have been assigned to locations based on their ordinal numbers. Our comparison revealed a decrease in cost of more than 10%.

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