GDP and Net Migration in Some Eastern and South-Eastern Countries of Europe. A Panel Data and Bayesian Approach

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panel data,
Bayesian test,
Central and Eastern European countries

ABSTRACT

This paper analyzes the relationship between the GDP and the net migration using the comparative approach represented by the panel data and Bayesian analysis. The panel data analysis is made for the period 1991-2013 in Central and Eastern European countries (CEECs): Czech Republic, Slovakia, Ukraine, Poland, Bulgaria, Romania, Croatia, Slovenia and Hungary. The Bayesian approach is utilized for checking whether Poland and Ukraine have similar expectation of GDP and net migration during 1991-2013. A negative correlation was recorded between net migration and the real GDP rate in the previous period. During the period of 1991-1994 corresponding to transition, each 10 percent increase in the GDP rate caused a 0.6 percent decrease in net migration at the significance level of 5%. A higher increase was registered in the economic depression (1994-2000), while the greater decrease, with 4.58 percentage points for each 10 percent in the GDP rate, was observed for the period of economic boom. Computing the differences between Ukraine and Poland, we obtained the expectations of -17.7187 and 15.745 with a higher variance of the estimator for Ukraine.

1. INTRODUCTION

Central and Eastern European countries (CEECs) record rising immigration, a process that puts the responsibility on their government to come up with the targeted immigration strategies for selecting a certain type of migrants. The economic literature attempts to study this problem looking at the immigrants’ absorption, their characteristics, and the consequences of their
presence in certain countries. However, few economists are interested in finding out the real determinants of international migration. With regard to this, it has to be noted that the trends in European migration in the last decade underwent a high number of spectacular changes.

A series of studies from economic literature analyzed migration developments and estimated the statistical relation between migration and its determinants (e.g. GDP rate, wage, unemployment rate, or productivity). The majority of these studies were based on the theories related the migration impact on labor markets of the target countries. It is considered that migration influences overall labor supply and wage in case of economies with fixed capital and small product range.

The main goal of our paper is to analyze the relationship between GDP and net migration in CEECs: Bulgaria, Romania, Croatia, Slovenia, Ukraine, Poland, Hungary, and the Czech Republic. Therefore, the panel data and the Bayesian approach were both applied to achieve comparative and compatible results. Our results make us to conclude that the economic prosperity influences immigration more than the unstable periods that are less favorable for immigrants in those specific countries.

The paper is structured in several sections. After the introduction, the two approaches (panel data and Bayesian approach) are described and the empirical results are presented. The last section concludes by giving important interpretations and discussion of results.

2. LITERATURE REVIEW: EFFECTS OF IMMIGRATION

The literature regarding the effects of immigration (there are many studies regarding to US and only few papers for Europe) follows various strands. Some of them provide predictions on the effects of migration on GDP and employment in a simulation framework supported by the Computable General Equilibrium (CGE) Models. The simulations consider competition between native labour and migrants, on various skill groups, considering mobility between industries and regions as an effect to immigration phenomenon, allowing the sector composition modification and supposing that demand consequences from the larger household sector that also include the immigrants.

Many studies analyzed the consequences of growth of immigration in convergence models. According to observations of Barro and Sala-i-Martin (2004), the effect of net migration on GDP per capita rate is small and positive, affecting the convergence parameter in a positive way. Etzo (2008) and Ozgen et al. (2010) offered surveys of the literature on this topic and Niebuhr et al. (2012) proposed an application for German economy. Barro and Sala-i-Martin (2004) discovered a negative impact of migration on GDP per capita growth. Ozgen et al. (2010) drew the conclusion that empirical applications usually proposed a small and positive effect with only one percentage point increase in migration rate when the GDP per capita increases only about 0.1 percent. Shioji (2001) explains that this is due to two countervailing effects migration. The quantity effect, referring to population growth, generates the reduction of growth and foster convergence. Moreover, the composition effect, referring to human capital composition of migrants, supposes a positive growth rate and economic divergence.

Clowes and Bilan (2014) having analyzed income per head trends in the Czech Republic, Hungary, Poland and Romania, it was revealed that higher GDP capita was a key factor in successful economic recovery of the countries during the global downturn.

Boeri and Brucker (2001) used an econometric model that predicted that the number of immigrants from the NMS in the Old Member States might increase from less than one million in 1998 to some three to 4.5 million people in 2030. Fassmann and Münz (2002), Čajka et al. (2014)conducted various macro and micro econometric studies for the estimation of migration flows from the East to the West. The functional form for the effect of GDP on migration is not
straightforward in literature. Pedersen et al. (2004) showed that the relationship is not necessarily linear, but in many cases it was observed a form of an inverted U-shape. Hatton and Williamson (2002) made a quantitative evaluation of the demographic and economic determinants of the migration in the entire world. The authors studied aspects like: the effect of poverty and inequality on migration, the distinction between ex-post migration and pressure of migration, the actuality of migration theories.

Jennissen (2003) studied the influence of economic factors on migration in the countries from Western Europe during 1960-1998. The dependent variable (net migration), is explained using exogenous variables like income per capita, education level and unemployment rate. The results based on regression models showed that the unemployment rate has a negative impact on net international migration.

Blanchflower (2007) showed that the immigrants to UK increased the supply more than the demand in short term. Moreover, it seems that the migration flow has advantages like a lower inflation pressure and a smaller natural unemployment rate. Longhi et al. (2008) showed that among 185 estimates of the effects of migration on unemployment analyzed in 13 studies, 81.6 % remain insignificant 12.4 % are significantly negative, if the probability is 95%. International migration may determine internal migration flows in the recipient nation. Therefore, the unemployment impact evaluation over the regions of the country can determine spurious positive effect of immigration on labour market.

Cadil and Kaderabkova (2012) evaluated the effect of migration on convergence among regions in the Czech Republic, discovering that there is no general relation between migration and GDP/capita or wages growth, neither positive, nor negative. 

Brunborg and Cappelen (2009) proposed an econometric model for migration in Norway, using as explanatory variables the unemployment rate in Norway and the income level in Norway relative to the average of OECD countries.

Ojapinwa (2012) determined the migrants’ remittances inflow considering as independent variable the GDP and other macroeconomic indicators. Passel, D’Vera Cohn and Gonzalez-Barrera (2012) made a description of the characteristics of the Mexican immigrants to USA.

Huber and Tondl (2012) analyzed the relationship between migration and regional convergence in European Union. It was made an econometric study to measure the impact of migration on regions from Europe and EU-27 during 2000-2007. The migration depends on output per capita and the labour productivity, but the regional perspective has not an influence on migration. In the European regions with low GDP, the migration does not determine convergence.

Bijak et al. (2013), Miłaszewicz et al. (2015), Bilan (2014), Minneci (2015), Oliveira et al. (2015) presented some scenarios regarding the future evolution of international migration in 27 countries from Europe during 2002-2052. The authors described the migration theories and simulations and predictions are made for net migration, labour force and population from selected European countries.

3. PANEL DATA APPROACH

Our empirical approach is in accordance with the recent researches made for several Eastern and Western countries. The recent studies provide evidence that there is no specific positive or negative relationship between GDP and migration. The relationship depends on the particular set of data. A comparative analysis based on econometric results for Eastern and South-Eastern Europe countries was not made before.

The variables are represented by the real GDP rate and the net migration (difference between the number of persons entering and leaving a country during the year per 1,000 people).
The period is 1991-2013 and the data refer to seven countries from Eastern and South-Eastern Europe: Ukraine, Poland, Bulgaria, Romania, Croatia, Czech Republic, Slovenia, Slovakia and Hungary. The dependent variable is the net migration and the independent one the GDP real rate. A positive value of the net migration shows that more people entering the country than leaving it, while a negative one implies that more people leaving than entering it. The real economic growth rate builds onto the economic growth rate by taking into account the effect that inflation has on the economy. The economic growth is concerned with the long-run trend in production due to structural causes such as technological growth and factor accumulation. We used annual data, so the problem of seasonality did not appear here.

The net migration is computed the same in other countries. The World Bank offers data sets with the values of net migration for almost all the countries, but this database has a short length. The start regression is the following one:

\[
\text{net}\_\text{migration}_{it} = c + b \cdot \text{GDP}\_\text{rate}_{t-1} + a_i + \epsilon_{it}
\]  

(1)

where:

- \( \text{net}\_\text{migration}_{it} \) — net migration in country \( i \) and year \( t \)
- \( \text{GDP}\_\text{rate}_{t-1} \) — real GDP rate in country \( i \) at time \( t-1 \)
- \( a_i \) — individual effects
- \( \epsilon_{it} \) — the error

First of all, we have to decide if we use a usual regression or a panel analysis. The OLS estimator is biased and inconsistent, the individual effects being present (Baltagi, 2008).

The application of Hausman test was made in order to decide if the model with fixed effects is better than the one with random effects or else. The probability associated to Hausman statistic is less than 0.05, the fixed effects model being better, according to results in Table 1. However, we have to take into account the economic reasons for this type of model.

The standard model used for the errors starts from two important assumptions: the errors are homoscedastic and non-auto-correlated. The results of the applied tests put into evidence auto-correlated and heteroscedastic errors. Therefore, some robust estimators are necessary.

**Table 1. Results of Hausman test**

<table>
<thead>
<tr>
<th>Test Summary</th>
<th>Chi-Sq. Statistic</th>
<th>Chi-Sq. d.f.</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross-section random</td>
<td>0.5420</td>
<td>1</td>
<td>0.4787</td>
</tr>
<tr>
<td>Cross-section random effects test comparisons:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Variable</td>
<td>Fixed</td>
<td>Random</td>
<td>Var(Diff.)</td>
</tr>
<tr>
<td>GDP_rate(-1)</td>
<td>0.1472</td>
<td>0.2564</td>
<td>0.0197</td>
</tr>
</tbody>
</table>

Cross-section random effects test equation:

<table>
<thead>
<tr>
<th>Dependent Variable: net_migration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross-sections included: 9</td>
</tr>
<tr>
<td>Total pool (balanced) observations: 180</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>-7.3489</td>
<td>3.8012</td>
<td>-1.9370</td>
<td>0.0520</td>
</tr>
<tr>
<td>GDP_rate(-1)</td>
<td>0.14657</td>
<td>0.0162</td>
<td>9.3345</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Source: own
For the two types of models (fixed effects model and random effects model), the hypotheses of errors’ homocedasticity and auto-correlation were checked and the results are presented in Table 2.

### Table 2. Results of the homoscedasticity and auto-correlation tests

<table>
<thead>
<tr>
<th>Tests:</th>
<th>Autocorrelation</th>
<th>Heteroscedasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed effects model</td>
<td>F(1,8)=5.32</td>
<td>Chi2(9)=16.919</td>
</tr>
<tr>
<td></td>
<td>F computed= 47.23</td>
<td>Chi2 computed= 35.18</td>
</tr>
<tr>
<td>Random effects model</td>
<td>F(1,8)=5.32</td>
<td>Chi2(9)=16.919</td>
</tr>
<tr>
<td></td>
<td>F computed= 88.44</td>
<td>Chi2 computed= 54.23</td>
</tr>
</tbody>
</table>

Source: own

Stata software package proved to be efficient in computing the regression with standard errors proposed by Driscoll and Kraay (1998), using the command xtscc. It is estimated the fixed effects model with Driscoll and Kraay errors that are heteroscedastic, auto-correlated up to a certain degree and possibly correlated with other groups. The results are presented in Table 3.

### Table 3. Estimations for fixed and random effects models (1991-2010)

<table>
<thead>
<tr>
<th>MODEL</th>
<th>Fixed effects model</th>
<th>Random effects model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>45.16 (18.502)*</td>
<td>53.39 (23.049)**</td>
</tr>
<tr>
<td>$\beta$</td>
<td>-0.00192 (0.0047)*</td>
<td>-0.036 (1.368)**</td>
</tr>
<tr>
<td>SIC (Schwarz information criterion)</td>
<td>463.223</td>
<td>562.784</td>
</tr>
</tbody>
</table>

Note: In brackets Driscoll-Kraay errors were written, the level of significance being 1% (*) and 10% (**) ;
Source: own

For the fixed effects model the coefficient is significant compared to the other model, that with random effects. Moreover, the SIC criterion has a lower value for fixed effects model.

The results for overall sample showed a negative correlation between net migration and real GDP rate when fixed effects model is used. Indeed, recent empirical researches put in evidence a decline in migration when a considerable increase was anticipated for GDP. We chose the fixed effects model because we have logical arguments: the statistical tests results are confirmed by the economic reality: there is correlation between the errors and the exogenous variable (real GDP rate in the previous period). Individual effects that affect the net migration (for example, the level of development of each country) is correlated with the GDP rate. An increase with one percentage point in average in the real GDP rate in the previous period will determine a decrease in the net migration with 0.19 percentage points. Therefore, we used three models corresponding to three periods: a) 1991-1994, b) 1995-2000, and c) 2001-2013.

The reasons for choosing these periods are related to some common aspects for the 7 countries. The period of a relative decline of GDP rate for these countries was interrupted by a relative growth in 1995-2000. Instead of b in each model we will have b1, b2 and, respectively b3. The models’ coefficients and the associated standard deviations that are written in brackets can be observed in Table 4.
Table 4. Coefficients and standard deviations for fixed and random effects models corresponding to the three periods

<table>
<thead>
<tr>
<th>Period</th>
<th>Fixed effects model</th>
<th>Random effects model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coefficient (std dev)</td>
<td>Coefficient (std dev)</td>
</tr>
<tr>
<td>1991-1994</td>
<td>-0.006 (51.22)**</td>
<td>0.012 (387.15)***</td>
</tr>
<tr>
<td>1994-2000</td>
<td>-0.036 (62.018)**</td>
<td>-0.0901 (114.65)***</td>
</tr>
<tr>
<td>2000-2013</td>
<td>-0.0458 (86.22)***</td>
<td>-0.742 (13.98)***</td>
</tr>
</tbody>
</table>

Note: *p<1% **p<5%, ***p<10%;
Source: own

A negative correlation can be observed between net migration and the real GDP rate in the previous period. For a level of significance of 5%, in the period 1991-1994 corresponding to transition, each increase with 10 percentage points in the GDP rate determined a decrease in net migration with 0.6 percentage points. A higher increase was registered in the economic depression (1994-2000), while the greater decrease, with 4.58 percentage points for each 10 percent change in the GDP rate, was observed for the period of economic boom. These results make us to conclude that the economic prosperity influences more the migration than the unstable periods are less favorable for immigrants in that specific country.

We take the case of Ukraine in order to estimate a single regression model with a dummy variable, corresponding to different periods (1995-2000 and 2001-2013). There are not statistically significant and economically reasonable results for Ukraine regression in 1991-1994. The dummy variable is called “period” and it takes the values 0 for the first period and 1 for the second one. For the last period we do not have to consider another value of the dummy variable. We have enough evidence from the graph to identify a correlation between net migration and those periods. Moreover, the correlation between GDP and the economic cycles is obvious.

We will consider the following regression model for Ukraine:

\[ \text{net_migration}_t = a_1 + a_2 \cdot \text{GDP\_rate}_{t-1} + a_3 \cdot \text{period}_t + \epsilon_t \]  \hspace{1cm} (2)

The GDP rate and the dummy variable are not correlated with the errors. The matrix of correlation is computed, observing coefficients of correlation closed to zero (0.0354, respectively 0.115). The data series are stationary. Taking into account that we have few data for Ukraine, the coefficients are estimated by stochastic simulations (bootstrap procedure). The number of replications is 10 000, the residuals being re-sampled. The Breusch-Godfrey Serial Correlation test for a level of significance of 5% has the test statistic equaled to 5.415, which is less than the critical value of 5.979. Therefore, the errors are independent. Moreover, the probability associated to White test is less than 0.05, fact that implies that the errors are homoscedastic. The results of these tests are presented in Table 5. The normality of the errors is assured by the large number of replications.

Table 5. Tests for serial correlation of order 1 and homoscedasticity

<table>
<thead>
<tr>
<th>Test</th>
<th>Computed statistics</th>
<th>Critical values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Breusch-Godfrey Serial Correlation test</td>
<td>5.4225</td>
<td>5.9889</td>
</tr>
<tr>
<td>Heteroskedasticity Test: White test</td>
<td>12.5965</td>
<td>15.8026</td>
</tr>
</tbody>
</table>

Source: own
The assumption of the regression model being checked, the following model was estimated, the parameters being statistically significant:

\[
\text{net\_migration} = 58.23 - 0.71 \cdot \text{GDP}_{t-1} - 0.462 \cdot \text{period}_t + e_t
\]  (3)

For Ukraine separately analyzed the sensitivity of the net migration to the previous GDP rate coefficient is higher. Higher GDP determined lower migration abroad. An explanation could be that the economic growth determines Ukrainian people to remain in Ukraine and also attracts foreign here.

4. BAYESIAN APPROACH

Let us consider \( X \) a random variable whose distribution depends on the parameter \( \theta \in \Theta \). If \( X \) is discrete random variable, we denote by \( p(x|\theta_0) = P(X = x|\theta = \theta_0) \), and, if \( X \) is continuous random variable, by \( f(x|\theta_0) \) the pdf of \( X \) that is conditioned by \( \theta = \theta_0 \).

In the Bayesian inference the parameter \( \theta \) is considered a random variable having the pdf \( g \), called the prior distribution of \( \theta \). The posterior pdf of \( \theta \) is:

\[
g(\theta|x) = \frac{p(x|\theta_0) \cdot g(\theta_0)}{\int_{\Theta} p(x|\theta) \cdot g(\theta) d\theta}
\]  (4)

in the discrete case, respectively:

\[
g(\theta|x) = \frac{f(x|\theta_0) \cdot g(\theta_0)}{\int_{\Theta} f(x|\theta) \cdot g(\theta) d\theta}
\]  (5)

in the continuous case.

According Preda (1992), if \( X \sim N(\theta, \sigma^2) \) with known \( \sigma^2 \) and prior distribution of \( \theta \) is \( N(\mu, \tau^2) \), then the posterior distribution of \( \theta \) is:

\[
N\left(\frac{\tau^2 \cdot X + \sigma^2 \cdot \mu}{\sigma^2 + \tau^2}, \frac{\sigma^2 \cdot \tau^2}{\sigma^2 + \tau^2}\right)
\]  (6)

It is also presented (see Preda, 1992) that a two-sided significance test in this case should yield the following results. We test the null hypothesis \( H_0 : \theta = \theta_0 \) with the first degree error \( \epsilon \) against the alternative hypothesis \( H_1 : \theta \neq \theta_0 \).

We consider \( p_0 \) the prior probability to have \( \theta = \theta_0 \), and, using this value we built a prior distribution having the cdf.

\[
\tilde{G}(\theta) = \begin{cases} 
(1 - p_0) \cdot G(\theta), & \text{if } \theta \leq \theta_0 \\
 p_0 + (1 - p_0) \cdot G(\theta), & \text{if } \theta \leq \theta_0
\end{cases}
\]  (7)

where \( G \) is a continuous cdf.

It has to be noted that the cdf \( \tilde{G} \) is continuous in all values of \( \theta \) except \( \theta_0 \), where it has a jump equal \( p_0 \). Usually, \( p_0 \) is taken 0.5, from maximum entropy principle (Preda, 1992).

The posterior probability to have \( \theta = \theta_0 \) is
\[ P(\theta = \theta_0 | X) = \frac{p_0 \cdot f(x | \theta_0)}{p_0 \cdot f(x | \theta_0) + (1 - p_0) \cdot \int f(x | \theta) \cdot g(\theta) d\theta} \] (8)

We accept \( H_0 \) if and only if the above posterior probability is at least \( 1 - \epsilon \).

In Ciuiu (2013) we use an estimator of \( \theta, \hat{\theta} \), instead of \( X \), and the above relation is rewritten as

\[ P(\theta = \theta_0 | \hat{\theta}) = \frac{p_0 \cdot g(\theta_0 | \hat{\theta})}{p_0 \cdot g(\theta_0 | \hat{\theta}) + (1 - p_0) \cdot g(\theta_0)} = \frac{p_0}{p_0 + (1 - p_0) \cdot R(\theta_0, \hat{\theta})} \] (9)

where

\[ R(\theta_0, \hat{\theta}) = \frac{g(\theta_0)}{g(\theta_0 | \hat{\theta})} \] (10)

In the case of normal distribution with known variance \( \sigma^2 \), in Ciuiu (2013) there is denoted \( \delta = \frac{1}{\sigma^2} \), and it is considered \( \theta_0 = 0 \). The above value of \( R \) becomes

\[ R_\delta(\hat{\theta}, \mu, \tau^2) = \frac{1}{\sqrt{\delta \cdot \tau^2 + 1}} \cdot \exp \left\{ \frac{(\delta \cdot \tau^2 \cdot \hat{\theta} + \mu)^2}{2 \cdot \tau^2 \cdot (\delta \cdot \tau^2 + 1)} - \frac{\mu^2}{2 \cdot \tau^2} \right\} \] (11)

Ciuiu (2013) runs the left-sided test (where the alternative hypothesis is \( H_1 : \theta < 0 \)) and the right-sided test (where the alternative hypothesis is \( H_1 : \theta > 0 \)). There are considered as prior distribution either the left (right) truncated normal distribution, or distribution such \(-\theta(\theta)\) is exponential. It is proved that the posterior distribution is left (right) truncated. More exactly, in the case of left-sided test, we have:

\[ g(\theta) = \frac{\varphi\left(\frac{\theta - \mu}{\tau}\right)}{\tau \cdot \Phi\left(-\frac{\mu}{\tau}\right)} \]

\[ g(\theta | \hat{\theta}, \delta) = \frac{\sqrt{\delta \cdot \tau^2 + 1}}{\tau} \cdot \frac{\varphi\left(\frac{\sqrt{\delta \cdot \tau^2 + 1}}{\tau} \cdot \left(\theta - \delta \cdot \tau^2 \cdot \hat{\theta} + \mu\right)\right)}{\Phi\left(-\frac{\delta \cdot \tau^2 \cdot \hat{\theta} + \mu}{\tau \cdot \sqrt{\delta \cdot \tau^2 + 1}}\right)} \] (12)

in the case of left-truncated distribution, and

\[ g(\theta) = \lambda \cdot e^{\lambda \cdot \theta} \]

\[ g(\theta | \hat{\theta}, \delta) = \sqrt{\delta} \cdot \frac{\varphi\left(\sqrt{\delta} \cdot \left(\theta - \hat{\theta} - \frac{\lambda}{\delta}\right)\right)}{\Phi\left(\sqrt{\delta} \cdot \left(-\hat{\theta} - \frac{\lambda}{\delta}\right)\right)} \] (13)
where $\lambda > 0$, in the exponential case.

In the case of right-sided test, we have

$$
\left\{
\begin{array}{l}
g(\theta) = \frac{\varphi\left(\frac{\theta - \mu}{\tau}\right)}{\tau \cdot \Phi\left(\frac{\mu}{\tau}\right)} \\
g(\theta|\hat{\theta}, \delta) = \frac{\sqrt{\delta \cdot \tau^2 + 1}}{\tau} \cdot \frac{\varphi\left(\frac{\sqrt{\delta \cdot \tau^2 + 1}}{\tau} \cdot \left(\theta - \frac{\delta \cdot \tau^2 \cdot \hat{\theta} + \mu}{\delta \cdot \tau^2 + 1}\right)\right)}{\Phi\left(\frac{\delta \cdot \tau^2 \cdot \hat{\theta} + \mu}{\tau \cdot \sqrt{\delta \cdot \tau^2 + 1}}\right)}
\end{array}
\right.
$$

(14)

in the case of right-truncated distribution, and

$$
\left\{
\begin{array}{l}
g(\theta) = \lambda \cdot e^{-\lambda \cdot \theta} \\
g(\theta|\hat{\theta}, \delta) = \sqrt{\delta} \cdot \frac{\varphi\left(\frac{\sqrt{\delta \cdot \left(\theta - \hat{\theta} - \frac{\lambda}{\delta}\right)}\right)}{\Phi\left(\frac{\sqrt{\delta \cdot \left(\hat{\theta} - \frac{\lambda}{\delta}\right)}\right)}
\end{array}
\right.
$$

(15)

where $\lambda > 0$, in the exponential case.

These tests were performed for the regression coefficients of the household savings in terms of nominal average wages, and the example is from Jula and Jula (2012), where the Student test is performed.

Solving the equation $P_\delta(0, 0, \tau^2) = 1 - \epsilon$ we obtain first the solution $\tau^2 = \tau_0^2$. Next, for $\tau^2 < \tau_0^2$ we solve the equation $P_\delta(0, \mu, \tau^2) = 1 - \epsilon$, and we obtain the solution $\mu = \mu_{\min}$ if $\hat{\theta} > 0$ or in the case of right-sided test and right-truncated normal distribution, respectively $\mu = \mu_{\max}$ if $\hat{\theta} < 0$ or in the case of right-sided test and right-truncated normal distribution (Ciuiu, 2013). The condition to accept the null hypothesis if $\hat{\theta} = 0$, and to reject it if $\hat{\theta} = \mu$ (a natural assumption) is to choose $\mu > \mu_{\min}$, respectively $\mu < \mu_{\max}$. In the exponential case we solve the equation $P_\delta(0, \lambda) = 1 - \epsilon$, and we obtain the solution $\lambda = \lambda_{\max}$. The condition to accept the null hypothesis if $\hat{\theta} = 0$ is to take $\lambda < \lambda_{\max}$. Finally we estimate the interval of confidence for $\hat{\theta}$ as the interval in which we must have the value of the estimator such that we accept $H_0$. Using this interval and the normal distribution, there is estimated the power of the test.

In the case of unknown variance (hence unknown $\delta$) we consider the prior distribution of $\delta$ as $\Gamma(a, b)$, obtaining the posterior distribution $\Gamma\left(a + \frac{m}{2}, \frac{2 \cdot b}{m \cdot b \cdot S^2 + 2}\right)$, where $S^2$ is an estimator of the variance of $\hat{\theta}$ with $m$ degrees of freedom (Ciuiu, 2013). $P_\delta$ is replaced by a value $P$ that does not depend on $\delta$: the expectation of $P_\delta$ according the above posterior distribution of $\delta$. The same expectation we do for the power of the test:
Consider the net migration and the real GDP for Poland and Ukraine in the period 1991-2013. First of all we test if the two countries have the same expectation in the case of the two above variables.

We compute the differences between Ukraine and Poland, obtaining the expectations -17.7187 and 15.745, and the variances of the estimators (variances divided by 23) 640.37911 and 32.05437.

First we consider known variances equal to the above estimators. For the net migration the results are listed in Table 6.

The powers of the test in Table 6 and Table 7 are calculated if the real value of the parameter is null, if its real value equals the right limit of the acceptance interval and if the real value equals the estimated value.

### Table 6. The Bayes tests in the case of net migration, known variance- Poland

<table>
<thead>
<tr>
<th>$\tau_0^2 / \lambda_{\text{max}}$</th>
<th>$\tau^2 / \lambda$</th>
<th>Type of the test</th>
<th>$\mu &lt; \mu_{\text{max}}$</th>
<th>Probability of accepting $H_0$</th>
<th>Interval for accepting $H_0$</th>
<th>Power of the test</th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td>$\tau^2 = 230000$</td>
<td>Two-sided</td>
<td>-24&lt;-23.14854</td>
<td>0.93692</td>
<td>(-0.27412, 0.27412)</td>
<td>0.99136</td>
</tr>
<tr>
<td>-</td>
<td>$\tau^2 = 230536.478$</td>
<td>Left-sided</td>
<td>-1&lt;-0.73681</td>
<td>0.93608</td>
<td>(-0.01318, 0)</td>
<td>0.99324</td>
</tr>
<tr>
<td>-</td>
<td>$\tau^2 = 230536.478$</td>
<td>Two-sided</td>
<td>-60&lt;-55.78763</td>
<td>0.93721</td>
<td>(-3, 3)</td>
<td>0.99324</td>
</tr>
<tr>
<td>-</td>
<td>$\tau^2 = 230536.478$</td>
<td>Left-sided</td>
<td>-9&lt;-8.26718</td>
<td>0.937</td>
<td>(-0.44996, 0)</td>
<td>0.99324</td>
</tr>
<tr>
<td>-</td>
<td>$\tau^2 = 250000$</td>
<td>Two-sided</td>
<td>-6&lt;-5.06877</td>
<td>0.93818</td>
<td>(-5.06899, 5.06899)</td>
<td>0.92606</td>
</tr>
<tr>
<td>-</td>
<td>$\tau^2 = 250000$</td>
<td>Left-sided</td>
<td>-1&lt;-0.66572</td>
<td>0.91257</td>
<td>(-1.92383, 0)</td>
<td>0.92606</td>
</tr>
</tbody>
</table>

Source: own
Table 7. The Bayes tests in the case of real GDP, known variance - Poland

<table>
<thead>
<tr>
<th>$\tau_0^2 / \lambda_{\text{max}}$</th>
<th>$\tau^2 / \lambda$</th>
<th>Type of the test</th>
<th>$\mu &lt; \mu_{\text{max}}$</th>
<th>Probability of accepting $H_0$</th>
<th>Interval for accepting $H_0$</th>
<th>Power of the test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\tau^2 = 11000$</td>
<td>Two-sided</td>
<td>23 &gt; 22.95</td>
<td>0.28009</td>
<td>(-0.03747, 0.03747)</td>
<td>0.99472</td>
</tr>
<tr>
<td>$\tau_0^2 = 11539.57455$</td>
<td></td>
<td>Left-sided</td>
<td>4 &gt; 3.29227</td>
<td>0.21047</td>
<td>(0.004102)</td>
<td>0.49741</td>
</tr>
<tr>
<td></td>
<td>$\tau^2 = 11539.57455$</td>
<td>Two-sided</td>
<td>15 &gt; 12.48139</td>
<td>0.28445</td>
<td>(-0.75, 0.75)</td>
<td>0.99724</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Left-sided</td>
<td>2 &gt; 1.73363</td>
<td>0.28611</td>
<td>(0, 0.1)</td>
<td>0.49295</td>
</tr>
<tr>
<td></td>
<td>$\tau^2 = 15000$</td>
<td>Two-sided</td>
<td>2 &gt; 1.11835</td>
<td>0.29005</td>
<td>(-1.11927, 1.11927)</td>
<td>0.99714</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Left-sided</td>
<td>0.5 &gt; 0.14488</td>
<td>0.23351</td>
<td>(0, 0.16211)</td>
<td>0.48858</td>
</tr>
</tbody>
</table>

Source: own

For instance, in the case of known variance, the first accepting interval is (-0.27412, 0.27412). The power of the test is 0.99136 if the real expectation is -0.27412 and the test power is the same for a real expectation of 0.27412.

The highest powers of the test for net migration with known variance were obtained for the case when we have a two-tailed test with $\tau_0^2 = 230536.478$ and $\tau^2 = 230000$.

The highest powers of the test for real GDP with known variance were obtained for the case when we have a two-tailed test with $\tau_0^2 = 11539.57455$ and $\tau^2 = 11000$. For real GDP, in the case of known variance, the first accepting interval is (-0.03747, 0.03747). The power of the test is higher than in the case of net migration, being 0.99472 if the real expectation is -0.03747 and the test power is the same for a real expectation of 0.03747. In Table 8 and Table 9, Bayes tests are computed in the case of real GDP of Ukraine.
Table 8. The Bayes tests in the case of real GDP, unknown variance- Ukraine

<table>
<thead>
<tr>
<th>$\tau^2_0 / \lambda_{max}$</th>
<th>$\tau^2 / \lambda$</th>
<th>Type of the test</th>
<th>$\mu &lt; \mu_{max}$</th>
<th>Probability of accepting $H_0$</th>
<th>Interval for accepting $H_0$</th>
<th>Power of the test</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau^2_0 = 233103.82809$</td>
<td>$\tau^2 = 230000$</td>
<td>Two-sided</td>
<td>-60&lt;-55.52595</td>
<td>0.93642</td>
<td>(-1.03661, 1.03661)</td>
<td>0.96673</td>
</tr>
<tr>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>0.96676</td>
</tr>
<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>0.97458</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Left-sided</td>
<td></td>
<td></td>
<td>(-0,0)</td>
<td>0.5</td>
</tr>
<tr>
<td>$\tau^2_0 = 233103.82809$</td>
<td>$\tau^2 = 233103.82809$</td>
<td>Two-sided</td>
<td>-57&lt;-56.06537</td>
<td>0.93678</td>
<td>(-2.81232, 2.81232)</td>
<td>0.90563</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.90638</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Left-sided</td>
<td></td>
<td></td>
<td>(-0,0)</td>
<td>0.5</td>
</tr>
<tr>
<td>$\tau^2_0 = 233103.82809$</td>
<td>$\tau^2 = 240000$</td>
<td>Two-sided</td>
<td>-5&lt;-4.27786</td>
<td>0.93746</td>
<td>(-4.27802, 4.27802)</td>
<td>0.86337</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.8655</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Left-sided</td>
<td></td>
<td></td>
<td>(-0,0)</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Source: own

The highest powers of the test for net migration with unknown variance were obtained for the case when we have a two-tailed test with $\tau^2 = 233103.82809$ and $\tau^2 = 230000$.

The power of the test is lower than in the case of known variance, being 0.96673 if the real expectation is -1.03661 and 0.96676 for a real expectation of 1.03661.

The highest powers of the test for real GDP with unknown variance were obtained for the case when we have a two-tailed test with $\tau^2 = 11815.32174$ and $\tau^2 = 11000$.

Table 9. The Bayes tests in the case of net migration, unknown variance- Ukraine

<table>
<thead>
<tr>
<th>$\tau^2_0 / \lambda_{max}$</th>
<th>$\tau^2 / \lambda$</th>
<th>Type of the test</th>
<th>$\mu &lt; \mu_{max}$</th>
<th>Probability of accepting $H_0$</th>
<th>Interval for accepting $H_0$</th>
<th>Power of the test</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau^2_0 = 11815.32174$</td>
<td>$\tau^2 = 11000$</td>
<td>Two-sided</td>
<td>30&gt;28.04654</td>
<td>0.29021</td>
<td>(-0.49315, 0.49315)</td>
<td>0.92978</td>
</tr>
<tr>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>0.93007</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>0.99805</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Left-sided</td>
<td></td>
<td></td>
<td>(-0,0)</td>
<td>0.5</td>
</tr>
<tr>
<td>$\tau^2_0 = 11815.32174$</td>
<td>$\tau^2 = 11815.32174$</td>
<td>Two-sided</td>
<td>26&gt;25.6229</td>
<td>0.29564</td>
<td>(-1.28243, 1.28243)</td>
<td>0.90584</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.90649</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Left-sided</td>
<td></td>
<td></td>
<td>(-0,0)</td>
<td>0.5</td>
</tr>
<tr>
<td>$\tau^2_0 = 11815.32174$</td>
<td>$\tau^2 = 12000$</td>
<td>Two-sided</td>
<td>1&gt;0.70263</td>
<td>0.29782</td>
<td>(-0.8264, 0.8264)</td>
<td>0.88398</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.88263</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Left-sided</td>
<td></td>
<td></td>
<td>(-0,0)</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Source: own
The power of the test is lower than in the case of known variance, being 0.49315 if the real expectation is -0.49315 and 0.93007 for a real expectation of 0.49315.

5. CONCLUSIONS

We analyzed the influence of the GDP dynamics on the net migration for seven CEE countries in the period of 1991-2013. The general results of our analysis showed that the increased income, where GDP is used as a proxy (Napolitano and Bonasia, 2009), has a negative influence on the net migration, which is calculated as a difference between the entering and leaving number of persons. This result appeared to be true both for models with fixed and random effects.

Our main result become seven more interesting if one pays attention to the details. The proposed model with assumed fixed effect provided better results in all specifications than the model with random effects. Statistically significant results were observed also when the sample was divided into three different time periods. GDP rate has negative influence on the net migration dynamics during the assumed periods of the transition, depression and economic boom in the Eastern Europe. In addition, the results we got suggest that the periods of economic prosperity (here, ‘economic boom’ in 2000-2010) influence more net migration than in the other periods, making it lower with each addition percent of the GDP growth, which is not intuitively anticipated. For Czech Republic and Slovakia, there is a negative and significant relationship between GDP and net migration. This conclusion was also obtained by Strielkowski (2012). In these countries the main reasons for emigration had a political and economic explanation. Many people felt free after the totalitarian regimes, while for some of them the standard of living was still unsatisfactory.

For Ukraine separately analyzed the sensitivity of the net migration to the previous GDP rate coefficient is higher. Higher GDP determined lower migration abroad. Consider the net migration and the real GDP for Poland and Ukraine in the period 1991-2013. First of all we test if the two countries have the same expectation in the case of the two above variables. Computing the differences between Ukraine and Poland, we obtained the expectations -17.7187 and 15.745, and the variances of the estimators 640.37911 and 32.05437.

Separate investigation for Ukraine supported the achieved results, when each next year and positive GDP trends make fewer Ukrainians leave the country and more Ukrainians to come back.

These unexpected results need more attention. In particular we have to consider a quality study of the migration dynamics and reasons for this reason. Also we have to consider other, unobserved factors like economic integration into EU of the Eastern European countries.
REFERENCES


Minneci, F. (2015), “If there were a ‘Highly Skilled Red Octopus’? The Case of Italian Highly Skilled Mobility at Times of Crisis”, Economics & Sociology, Vol. 8, No 3, 170-182. DOI: 10.14254/2071-789X.2015/8/3/13


